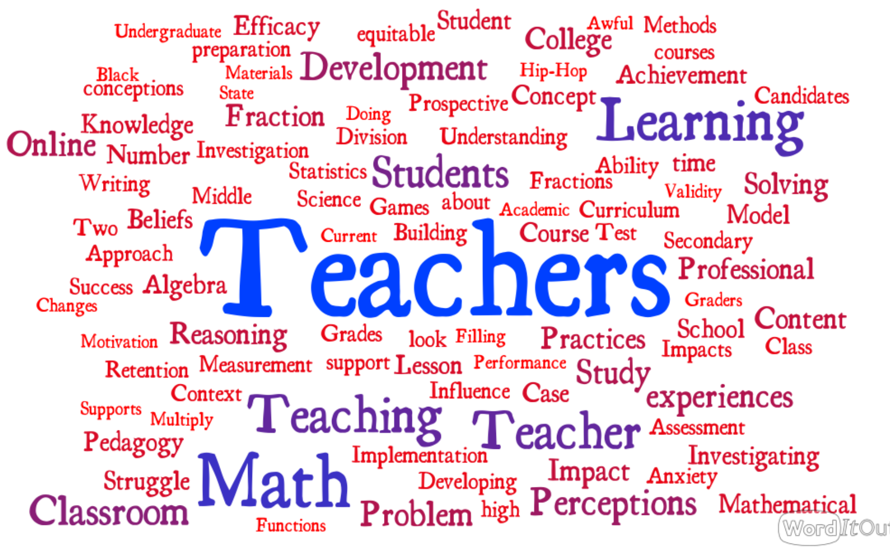
RCML

Proceedings for the 45th Annual Meeting of the Research Council on Mathematics Learning

Let the Good Times Roll in Mathematics Learning



Baton Rouge, LA
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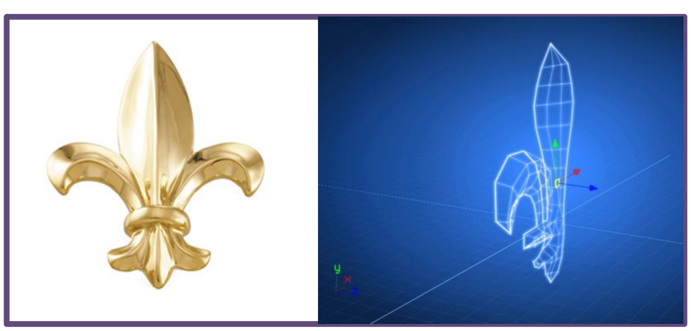

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Please Note:

RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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IMPACT OF MATHEMATICS PROGRESSIONS ON TEACHER CONTENT KNOWLEDGE

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This paper presents a comparison from three state-level Mathematics and Science Partnership Projects focused on the development of elementary and middle school mathematics teachers' content knowledge. Changes in content knowledge were measured using the Content Knowledge for Teaching Mathematics Assessment, developed by the Learning Mathematics for Teaching Project at the University of Michigan. Initial findings indicate that focusing on the progression of mathematics learning may have a greater impact on teachers' mathematics content development than a grade-level or domain-specific focus.

Through multiple iterations of state-level Mathematics and Science Partnership (MSP) projects, our research team has gained valuable insight into the development of quality training programs for inservice elementary and middle school mathematics teachers. With each successive project, we adjusted the design of the professional development (PD) experience to reflect lessons learned from previous projects. In this paper, we share how these changes across projects show an impact on the development of the mathematics learning for inservice teachers.

Participating teachers in these projects consisted of elementary and middle school classroom teachers. Classroom teacher assignments ranged from lower elementary fully- contained classrooms through middle school subject-specific classrooms. The majority of project participants came from high-need, rural school settings.

The project team made decisions regarding the content focus of teacher training programs using information from national research publications, national standards initiatives, and state-level curriculum documents. In our most recent teacher PD project, the focus on the progression of content knowledge across elementary and middle school grades had a significant impact on the content knowledge development of participating teachers. While initial projects showed positive impacts on teacher content knowledge, the narrower content focus did not produce the significant results that were shown from the focus on the progression of mathematics across grade levels.

Theoretical Framework

Since the implementation of more rigorous content standards in mathematics, there is a need for additional content development of teachers. The deep conceptual nature of these new standards goes far beyond the mathematical training of many elementary and middle school teachers. Expanding teachers' current mathematical understandings requires them to be engaged in transformative PD designed to change the way they think about how mathematics should be taught (Smith, 2001). Mathematics learning should involve opportunities to make sense of problems and reason through solution processes (NCTM, 2000; NRC, 2001). By engaging in these practices, teachers will experience the impact that these practices can have on students.

In developing teachers' content knowledge, PD must engage teachers in collaboratively completing tasks that enable them to expand their views of what it means to know and understand mathematics (NRC, 2001). The type of mathematical knowledge required to facilitate learning represents a special type of content knowledge (Ball, Thames, & Phelps, 2008) and the development of this content knowledge is enhanced by participating as a learner in a classroom environment in which the instructor models appropriate instructional techniques and behaviors (Loucks-Horsley, Hewson, Love, & Stiles, 1998; Smith, 2001). This transformative approach to PD must be presented through multiple, consistent opportunities for teachers to engage in hands-on, active learning through tasks that are coherent and relevant to their classrooms (Tate & Rousseau, 2007) and should align with the mathematics that teachers are required to teach (Cohen, 2004), based on state frameworks.

According to Heritage (2008), "Learning is envisioned as a development of progressive sophistication in understanding and skills within a domain" (p. 4). Therefore, teachers should engage in tasks that facilitate their understanding of the development of the mathematics in earlier grade levels through later grade levels (Cohen, 2004). In a report by the Consortium for Policy Research and Education, the authors state, "Learning progressions can inform teachers about what to expect from their students. They provide an empirical basis for choices about when to teach what to whom" (Daro, Mosher, & Cocoran, 2011). Our study suggests that engaging teachers in exploring these learning progressions may also have a significant impact on their content knowledge for teaching mathematics.

Methodology

Funding provided by a state-level MSP program allowed for multiple projects across several years. With each iteration of funding, the structure of the PD remained the same with slight variations on the content focus, based upon lessons learned from earlier projects. Each project consisted of a two-week summer institute, three follow-up PD sessions during the academic year, and monthly online discussions.

The Content Knowledge for Teaching Mathematics (CKTM) Assessment, developed by the Learning Mathematics for Teaching (LMT) Project at the University of Michigan (Ball, Hill & Bass, 2005), was used as the primary assessment of pedagogical content knowledge. Assessment items were chosen from a bank of elementary and middle school mathematics items focused on the concepts of patterns, functions, and algebra; geometry; and numerical reasoning. Each item had a value of one point and was piloted by LMT with over 500 elementary teachers, yielding information about item characteristics and overall scale reliabilities for various piloted forms of the exam. To develop these assessments for each project, project goals and content focal areas were provided to external evaluators, who selected items from the CKTM item bank for the project team to review. The project team identified the items which were aligned with planned professional development activities. Project evaluators then used this feedback to develop the content assessments. The project-specific assessment was administered to participating teachers in each project as a pretest preceding the summer institute. The same instrument was administered again on the last day of the institute to the same individuals. The pretests and posttests for individuals were then paired so that individual progress could be examined. In the following sections, we will provide a brief description of each project as well as a comparison of the participating teachers.

Promoting Innovation in Mathematics Education (Project PrIME): 2009-2012

Project PrIME envisioned mathematics classrooms in which students were engaged in worthwhile mathematical tasks that afforded them the opportunity to learn mathematics through problem solving. Project participants included mathematics teachers from Grades 4-8 from schools across North Mississippi. Individual teachers were recruited from area schools with some schools and school districts having multiple teachers represented, while others had only one participating teacher representing the school.

The content focus for the summer institute of Project PrIME was teaching mathematics through problem solving. Teachers had the opportunity to participate in the project for multiple years. Using the description of problem solving and the content strands as outlined by the *Principles and Standards for School Mathematics* (NCTM, 2000), teachers had the opportunity to focus on a different content area for each year of participation. First-year participants focused on the number and operations and algebra strands; second-year on geometry; and third-year on measurement. For this study, we are only reporting on the content development of first-year participants, as that content was similar across all three MSP projects.

All teachers (Grades 4-8) participated together in one class exploring the same content, which allowed teachers to bring unique perspectives to the variety of problem solving strategies based on their grade level. Since the goal of the project was to develop teachers' understanding of teaching through problem solving, the actual content of numbers, operations, and algebra was used as a vehicle for presenting ideas related to problem solving.

The CKTM Assessment used for Project PrIME consisted of 33 items chosen from the patterns, functions, and algebraic reasoning bank of items. Matched pretests and posttests were analyzed for 103 first-year participants of Project PrIME.

Developing Excellence in Education through Professional Learning Communities (DEEPLC Project): 2012-2015

The DEEPLC Project aimed to meet teachers where they were in their school communities to develop cohorts of effective mathematics instructors who engaged students in developing a deep understanding of mathematics concepts and worked collaboratively with peers in meaningful professional learning communities. Project participants were recruited from area school districts as cohorts from within those districts. In order to be eligible to participate in the project, the district had to have a cohort that consisted of at least three mathematics teachers from Grades 4-8 and one administrator. The project team based this decision on observations from the previous project noticing that teachers who had colleagues who had also participated were more likely to implement instructional changes in the classroom. This evidence was based on observations of classroom instruction.

The DEEPLC Project utilized the *Common Core State Standards for Mathematics (CCSSM)* (National Governors Association, 2010) as its resource for content alignment, as Mississippi had adopted the *CCSSM* as its state-level mathematics curriculum in 2010. The goal of this project was for teachers to gain a deeper understanding of the content they taught as outlined by the *CCSSM*. For this purpose, teachers were separated into classes during the summer institute based on their grade-band, elementary (Grades 4-5) and middle level (Grades 6-8). The elementary class content focused on the numbers and operations and operations and algebraic thinking domains from the *CCSSM*, while the middle grades class content focused on the expressions and equations and algebra domains from the *CCSSM*. This division allowed teachers to explore deeply the content for which they were responsible in their classroom.

The CKTM Assessment for the DEEPLC Project utilized 45 items each for the elementary version and the middle school version. The assessment items were selected based on the content addressed during the summer institute which was determined by examining the critical areas of the *CCSSM* for each grade band. Matched pretests and posttests were analyzed for 64 elementary teachers and 53 middle school teachers (117 matched assessments over three years of the DEEPLC Project).

Creating Continuity and Connections across Content (C4 Project): 2015-2017

The C4 Project envisioned consistent and coherent mathematics instruction across kindergarten through 8th grade mathematics classrooms in which students were engaged in learning through problem-solving and teachers utilized ongoing formative assessment in their instructional practices. Recruitment began through an application process through which individual schools were asked to indicate their level of commitment to participation in the project and the number of teachers interested in participating. Individual mathematics teachers from Grades K-8 were then recruited from selected schools. There was no limitation to the number of participants from each school.

The content focus for the C4 Project was to develop a deep understanding of the progression of learning across the elementary and middle school grades with specific emphasis on numbers, operations, and algebraic reasoning as outlined by the current state standards. These content standards were the state-level adaptation of the *CCSSM*. Since this iteration of funding extended the grade level of participants from Grades 4-8 to Grades K-8, the project team was able to expand the content focus to the primary grades. Since the content focus was on understanding how mathematics content is developed across grade levels, all teachers (K-8) were together in one class during the instructional time for the two-week summer institute. This content focus allowed the project team to emphasize the connections of the mathematics from the primary grades, to the upper elementary grades, through the middle school curriculum.

The CKTM Assessment used for the C4 Project consisted of 31 items designed to capture teachers' mathematics content knowledge as well as how teachers solve the special mathematical tasks that arise in teaching, including evaluating unusual solution methods, using mathematical definitions, representing mathematical content to students, and identifying adequate mathematical explanations. Pretests and posttests were paired for 34 participants in Cohort 1 and 42 participants in Cohort 2 of the C4 Project (76 matched assessments over two years).

Table 1

Project Participant Distribution

	Number of schools	Number of school districts	Individual participants	School cohort participants	Total participants	Matched pretests and posttests (*n*)
PrIME	69	38	73	44	117	103
DEEPLC	18	9	4	117	121	117
C4 Project	17	9	1	85	86	76

Findings

These three projects measured similar content across four different versions of the CKTM Assessment. Therefore, the research team is reporting effect sizes as a means to compare growth in mathematics content knowledge across these groups. According to Cohen (as cited in Centre for Education Statistics and Evaluation, 2014), "Effect sizes are often used to measure the difference in performance of two groups" (pg. 2). Cohen suggested that 0.2 be considered a 'small' effect size, 0.5 a 'medium' effect size, and 0.8 a 'large' effect size.

Results for Project PrIME were analyzed for 103 teachers from Years 1, 2, and 3 combined who had both pretests and posttests which each contained 33 assessment items. There was almost a three-point (2.9) gain in the mean from pre administration to post administration. This difference between the pretest score arithmetic mean (15.94) and the posttest score arithmetic mean (18.03) yielded an effect size of 0.35. This index falls in the range for small effects, and meets the benchmark for changes in content knowledge set by the project. When the effect size is 0.35, the cumulative probability is 0.63, meaning the upper-half of the posttest score population exceeded 63% of the pretest score population.

Results for the DEEPLC Project were analyzed for 64 elementary teachers from the Years 1, 2, and 3 summer institutes who had both pretests and posttests which each contained 45 assessment items. There was over a 2-point (2.7) gain in the mean from pre- to post-administration. This difference between the pretest score arithmetic mean (26.69) and the posttest score arithmetic mean (29.39) yielded an effect size of 0.53. This index falls in the range for medium effects. Results were also analyzed for 53 middle school teachers from the Years 1, 2, and 3 summer institutes who had both pretests and posttests which each contained 45 assessment items. The point increase of the mean from pre to post administration was 2.6. This difference between the pretest score arithmetic mean (28.06) and the posttest score arithmetic mean (30.68) yielded an effect size of 0.40.

Results for the C4 Project were analyzed separately for each cohort of teachers since the project is currently in its second year of funding with the possibility of having a third year. For cohort 1, results of the 31-item assessments were analyzed for 34 teachers. The mean score at baseline was 12.32. The mean score at follow-up was 15.44. Participants showed more than three points (3.12) growth on the assessment. Using the pretest standard deviation of 5.1, the effect size is 0.611. For cohort 2, results of the same 31-item assessment were analyzed for 42 teachers. The mean score of the pretests was 13.23 with standard deviation of 5.30. The mean score of the posttests was 17.92 with standard deviation of 5.4. Participants showed a mean gain of 4.7 points on the assessment and the effect size was 0.883, within the range of large effects.

Table 2

Assessment Results

	Number of items	*n*	Pretest mean	Pretest SD	Posttest mean	Posttest SD	Effect size
PrIME	33	103	15.94	6.01	18.03	6.23	0.35
PD Content Focus: Teaching through problem solving.							
DEEPLC Elementary	45	64	26.69	5.08	29.39	4.86	0.53
DEEPLC Middle	45	53	28.06	6.52	30.68	6.45	0.4
PD Content Focus: Grade level specific mathematics standards.							
C4 Project cohort 1	31	34	12.32	5.1	15.44	4.89	0.611
C4 Project cohort 2	31	42	13.23	5.3	17.92	5.4	0.883
PD Content Focus: Progression of related skills from primary through middle grades curriculum.							

n represents number of participants within each project with matched pre- and post-assessments.

Discussion

The third project produced significant findings on the development of teacher content knowledge which may be linked to a focus on the progression of mathematics content across elementary and middle grades. While some of the same tasks were utilized from the first two projects, many new investigations into how the content was developed were included. For example, the content for the first week of the summer institute focused entirely on addition and subtraction from introduction of these concepts through counting in Grades K-1, through developing an understanding of place value and its connection to addition and subtraction algorithms for multidigit whole numbers in Grades 2-3, to connecting these foundational ideas to addition and subtraction of fractions in Grades 4-5, into applying these understandings to operations with integers in Grade 6 and ultimately to understanding the generalized arithmetic of expressions and equations in Grades 7-8. A similar progression was developed in week two with regard to multiplication and division. By exploring the full progression of a mathematical concept from introduction to algebraic generalization, teachers were able to gain a complete understanding of the concept and of where the content they teach fits within the trajectory of student learning. Further investigation of this idea may be beneficial to future studies on the impact of professional development for inservice mathematics teachers.

Acknowledgement

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CHARACTERIZING PROSPECTIVE MATHEMATICS TEACHERS' PRODUCTIVE STRUGGLE

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This exploratory case study examined prospective elementary, middle grades, and secondary teachers' (PSTs) perception of their struggles in a mathematics content course for teachers. In particular, this study considered the ways that the PSTs characterized their struggle while engaging in a non-routine problem-solving task. Results indicated that the PSTs attempted to describe their own struggles while they focused on getting the correct answer and making sense of their work.

Students' struggle to learn mathematics is often viewed as problematic, but research suggests that the right amount of struggle can be a productive and necessary component of learning mathematics (Hiebert & Grouws, 2007). In the field of mathematics education, more and more students are encouraged to engage in productive struggle to learn mathematics for understanding (NCTM, 2014; Warshauer, 2015). Productive struggle refers to the "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, p. 387). This idea that struggle can be productive is linked to students making sense of and persevering in solving problems (NCTM, 2014) and would suggest a need for a growth mindset (Star, 2015). Students with a growth mindset believe that abilities and intelligences can be developed as opposed to being fixed traits (fixed mindset) (Dweck, 2007). Effective mathematics teaching provides opportunities and supports for students to engage in productive struggle to understand mathematics at a deeper level rather than focusing on finding correct solutions (NCTM, 2014). To better understand the process of productive struggle and how best to engage prospective mathematics teachers in productive struggle (PS), a small exploratory case study was conducted in three mathematics content courses to examine (1) how prospective elementary, middle, and secondary teachers engaged in PS as they completed non-routine mathematical tasks and (2) how prospective teachers perceived and characterized their struggle during non-routine mathematical tasks.

Conceptual Framework and Related Literature

Two conceptual frameworks guiding this study are: (1) the role of struggle in learning mathematics with understanding, and (2) student self-reflection. The importance of students struggling to learn has a long history of proponents including Dewey's (1933) process of engaging students in some doubt or confusion, Piaget's (1960) restructuring their disequilibrium, Handa's (2003) linking struggle with mathematical engagement, and Hiebert and Wearne's encouraging all students to "struggle with challenging problems" to "learn mathematics deeply" (2003, p. 6). In their study, Warshauer, Herrera, Starkey, and Smith (2017) examined the development of preservice teachers' understanding of PS in a mathematics content course. The focus of their study was on developing teaching practices that 1) support PS in learning mathematics and 2) enhance professional teacher noticing to interpret student struggles and student understanding. Despite this, little research exists on what PS looks like for prospective elementary, middle grades, and secondary teachers. The current study adds to the research base by focusing on PS in prospective K-12 mathematics teachers' perception of their own PS in mathematics content courses and allowing them time to reflect on their struggle after they attempted a task. Moving PSTs to a deeper understanding by engaging them in reflection on class activities is an important aspect in teacher preparation with many proponents (e.g., Borko, 2004; Dewey, 1933; Schussler, Stooksberry, & Bercaw, 2010).

Methodology

Context (Productive Struggle in Mathematics Content Courses for Teachers)

This research study was conducted by four mathematics teacher educators (MTEs) from a mid-sized university in the Midwest. Three of the four MTEs were the course instructors and are referred to as course instructors from this point forward. These courses instructors teach content courses in the Mathematics Department. The fourth MTE teaches in the School of Teacher Education. Because we sought to understand the phenomenon of PS among preservice teachers within three different classes, we found that an exploratory case study was the most appropriate design (Creswell, 2013).

Participants

The participants were 32 prospective elementary, middle, and secondary teachers from three different mathematics content courses. Of the 24 students enrolled, only 21, three males and 18 females, were present and participated from Course A, a required course for elementary and middle grades majors with a focus on the conceptual development of geometry concepts. Five students, one male and four females, participated from Course B, an elective course for mathematics majors who are also seeking secondary teaching certification. Another student had stopped attending and was not present during data collection. Course B is a problem-solving course that addresses content from previous mathematics courses in which the students were enrolled. All six students, two males and four females, participated from Course C, a required course for middle and secondary majors that is an in-depth study of functions and mathematical topics used in teaching pre-calculus and transition-to-calculus courses mainly at the secondary level. The students in each class were required to engage in classroom discussions and to explain their thinking on all class activities and assignments.

Data Collection and Analysis

Our investigation into the role of PS in collegiate mathematics content courses was exploratory in nature. Each of the three courses represented a different case in the overall exploratory case study. The goal was to identify and characterize the nature of prospective teachers' struggles as they engaged in a non-routine mathematical task in each of the three cases. The four MTEs met prior to the lessons to choose, discuss, and plan the task (see Table 1) for each lesson. Each of the three course instructors taught one lesson, and each presented a different problem-solving activity. During the lesson, the other MTEs observed the lesson and took field notes on students' actions, discourse-based evidence of student thinking, and evidence and description of student struggle. Courses B and C were 80-minute classes that met twice a week. Course A met three days a week for 55 minutes. At the end of each of the lessons, the prospective teachers were asked to complete an exit ticket that consisted of the following three questions: 1) How would you characterize your struggle with the task(s) you worked on today? Be specific. 2) What interactions with your group helped you progress through the struggle? Be specific. 3) What tools did you use to help look for a solution? How did they help you? Be specific.

Once the observation notes and exit tickets were collected, each MTE coded the data individually using Warshauer's (2015) list of four types of student struggles: (1) getting started, (2) carry out a process, (3) uncertainty in explaining and sense-making, and (4) express misconception and errors as a guide to developing themes. To ensure the trustworthiness of the data analysis, the researchers involved in the study analyzed a sample of the exit ticket responses and observations individually, highlighting significant statements or quotes that provided an understanding of how the participants experienced the task, interpreting the meaning of the statements and developing potential themes (Creswell, 2013). After initial themes were developed separately, the researchers met to discuss and establish consensus on emerging themes before separately coding the rest of the data set for each question. Finally, the researchers met again to come to agreement on all responses based on the original set of themes that were developed. This process resulted in three categories: group discussion, dynamics, and function; teacher role as facilitator; and perseverance and struggle.

Table 1

Mathematical Tasks Descriptions

	Mathematical Task
A	*Pennies Problem Solving Task* Put 9 pennies heads-side-up in a 3x3 grid so that the edges of the pennies are touching. A move consists of choosing a square, turning over the penny in that square, and then turning over all pennies touching that first one. How many moves must you make and which square must you choose in order to finish with 9 tails-side-up pennies? What if we had a 4x4 grid? 5x5 grid? Is there a strategy for this game?
B	*Monkey & Coconuts Task* Three sailors and a monkey are shipwrecked on a deserted island with coconuts as the only source of food. The sailors collect coconuts but decide to divide them in the morning. During the night, one by one, each sailor divides the coconuts into three equal shares and gives the remaining one to the monkey. This happens three different times. In the morning, the sailors meet and once again divide the coconuts into three equal piles and again give the remaining coconut to the monkey. What is the least possible number of coconuts in the original pile?
C	*Roller Coaster Task* Build a rollercoaster consisting of a piecewise function that is differentiable over its entire domain $[0,15]$ with entrance and exit points at $(0,10)$ and $(15,0)$ respectively and a local minimum and maximum of $(4,2)$ and $(8,8)$ respectively.

Findings

PSTs identified both positive and negative types of struggle in relation to the tasks that they completed. Descriptions of frustration/confusion/disengagement or the problem as being too challenging were classified as negative struggle because we found that these struggles did not support students in reaching the goal of the task. Positive struggles included wanting more time, persevering, and group dynamics and were labeled as such because they did support students in either reaching or beginning to reach the goal of the task. A discussion with examples of the struggles identified by PSTs is described below.

Negative Struggle #1: Frustration/Confusion/Disengagement

Those who expressed frustration or confusion with the task described this confusion regarding how to begin solving the task ("The task had me looking for patterns which is a difficult concept for me when I don't know where to start. Before I was given direction, I struggled to even begin the task" in Course A). Other PSTs struggled initially but described how their classmates helped them to overcome the struggle ("When I was given the task to work, I was unsure where to start. Each group member had a different idea about what the problem was asking. Overall, I feel like with my group and our different ideas, it made it easier to work through the problem by putting all of our ideas together" in Course C).

Negative Struggle #2: Too Challenging

Some PSTs expressed difficulty implementing or carrying out a process ("I found I struggled grasping the concept at first. Making sure I was turning over the right coins. Remembering where I started was also something I struggled with" in Course A) or encountered an impasse ("Once I had given up on the problem, I did not really participate in any discussion, because I hadn't made sense of the problem myself, thus I couldn't give any feedback when my group was discussing. I feel that it takes me a lot longer to solve these problems we are given in class than my other classmates, so I tend to give up trying once everyone else has figured out the problem" in Course B). Some PSTs described how the groups helped them to overcome the struggle of carrying out a process ("Having different inputs helped me see different ways to go about the problems" in Course A).

Positive Struggle #1: Perseverance and More Time

During the group discussions, PSTs' experienced difficulty explaining their thinking ("The struggle that I found when working on the task was finding the patterns of each chip. I also struggled in trying to explain how I got my answer" in Course A). PSTs found that listening to classmates' reasoning helped them to make sense of the work ("Our group discussed what would be the best approach to the problem. Members of the group argued or defended their own ideas. Using each of our ideas, the given information and deciding which idea worked best was a major part of our discussion. We each worked out the problem and compared our work once we agreed on the best way to solve it. This assured us that we each knew what was going on while creating new and better ideas" in Course C). Many PSTs' wanted more time to explore the problems ("It was definitely challenging working through the problem but not knowing the answer. I wish we had more time to try new equations. We had some ideas as a group about different approaches we could take, but it was hard not knowing what direction we were meant to go in, or if we were on the right track or not" in Course C)

Positive Struggle #2: Group Dynamics

In their exit tickets, the PSTs reflected on the role of group members in their struggle with the tasks. For example, several PSTs identified group dynamics as having a positive impact on their struggles with the tasks ("By us all struggling with it and then putting our minds together and figuring it out, like one of the team members saying why don't we work from the outside in, so by us doing so we were successful" in Course A. "I would characterize my struggle with the task as a productive type of struggle. While the task was frustrating at times, the concept made myself and my group think critically about the content we had learned and how it could be properly applied to this new situation" in Course C).

Discussion and Implications

Our findings showed similarities to Warshauer's (2015, 2017) work, with two exceptions. First, we observed that the emerging themes encompassed more than what Warshauer's team initially represented. As noted in the results, the extending themes that surfaced in this study were (in italics):

- Getting started – *Initiating with the task*
- Carrying out process – *Continuing with the task*
- Uncertainty in explanation and sense-making – *Explaining reasoning*
- *Prospective Teacher Reflections*

With the further definition and a granular look at this angle of research, we found that the participants' struggle expanded beyond the work of the platform provided by Warshauer by specifying more refined aspects of "getting started," "carrying out the process," and "uncertainty in explanation and sense-making."

Second, we noticed stark differences among the responses from PSTs in the elementary course in comparison to the other two courses. Specifically, we noticed differences in comfort level and passion for mathematics among the groups. Among the elementary majors, there was a higher occurrence of students with discomfort regarding mathematical tasks that initiated mathematical struggle than with the middle grades and secondary mathematics group. We hypothesize that this might be due to the self-sorting that happens by students choosing what major or career path to take: A difference within Course A is that some of the participants are dual certification candidates of Elementary and Special Education, further showing passion for helping children of need, but also potentially giving us a glimpse into some of their own personal struggles as to why they chose to be teachers. Additionally, we wonder if the increased negativity in responses from those in Course A was due to a portion of the activity not having a solution.

In Courses B and C, the middle and secondary mathematics education teachers may very well tend to choose mathematics teaching due to positive experiences in mathematics class, feel a confidence with their skills, and have a better understanding of the content. This was evident among the responses from PST in Courses B and C as they tended to be more positive. A potential anomaly that our team has wondered about is whether the students in the middle/secondary mathematics courses could potentially have self-selected also by self-efficacy, math content knowledge, and abilities. In other words, in contrast, the elementary and special education majors could be viewing themselves as generalists and not seeing the need to be "experts" with the similar skill-set we are pushing all mathematics content course participants to have.

Although these results come from a small case study, and as such, are not necessarily generalizable to a large population, we believe these two differences from Warshauer's (2015, 2017) work offer new insight into the role of struggle in mathematics for PST. Specifically, these results seem to indicate that particular attention needs to be paid in supporting elementary PST struggle productively in their mathematics content courses.

As this group of PSTs, overall, had more negative reflections on their struggle with mathematics, it is imperative that we, as mathematics teacher educators, help this population develop the tools and mindset needed to approach challenging mathematical tasks in productive ways.

Future Research

To get at the differences between the groups of preservice teachers—elementary, middle, secondary—a potential implication for research would be to do follow-up research or add to the research regimen a step that includes asking the three different groups why they perceive their struggle in mathematics so differently. By asking this question, we may gain insight also into the perception of one group of teachers compared to another – potentially leading to another uniquely and culturally impactful study that could impact school climate and mathematics learning research.

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MATHEMATICS TEACHER EFFICACY AND MATHEMATICS ANXIETY IN PRESERVICE TEACHERS

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This study investigated the relationship between mathematics anxiety and mathematics teacher efficacy among 347 elementary preservice teachers from a southeastern United States university. Data sources included the Mathematics Anxiety Rating Scale (MARS), Mathematics Teaching Efficacy Beliefs Instrument, and preservice teacher interviews. Findings revealed a significant, negative relationship between mathematics anxiety and mathematics teachers' efficacy. Specifically, the preservice teachers with the lowest degree of mathematics anxiety had the highest levels of mathematics teacher efficacy. Interviews with preservice teachers indicated that their mathematics anxiety is associated with efficaciousness toward mathematics teaching practices and is the basis for their mathematics teaching efficacy beliefs.

Theoretical Framework/Related Literature

Mathematics educators actively seek to develop and design programs that effectively prepare preservice teachers including efforts to reduce mathematics anxiety and enhance mathematics efficacy beliefs. An important goal of every teacher education program should be to help preservice teachers develop beliefs and dispositions that are consistent with current educational reforms and to assist them in addressing their mathematics anxiety (Gresham, 2017). Responding to the prevalence of mathematics anxiety among preservice teachers and its implications for mathematics teaching effectiveness, studies exist that examine the relationship of mathematics anxiety to other constructs. Although there are studies concerning mathematics anxiety in preservice teachers and separate studies on teacher efficacy, there is limited research regarding preservice teachers' mathematics anxiety and its relation to mathematics teacher efficacy, specifically with elementary preservice teachers.

Preservice teachers who experience mathematics anxiety are more likely to have negative views of mathematics and teach in ways that develop mathematics anxiety in their students (Bekdemir, 2010). Mathematics anxiety as well as low mathematics self-efficacy and mathematics teachers' efficacy beliefs also affect teacher engagement and practice in their future classrooms (Taylor & Fraser, 2013). The instructional methods and pedagogical approaches used by college instructors have the greatest impact on shaping preservice teachers' attitudes and views toward mathematics (Nisbet, 2015). In addition, mathematics methods courses that integrated reform-based constructivist methods, including hands-on activities and inquiry-based strategies, successfully reduced preservice teachers' mathematics anxiety levels (Sloan, 2010).

Mathematics anxiety is defined as an irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations and is considered much more than a dislike toward mathematics (Tobias, 1998). Tobias further describes mathematics anxiety is the lack of comfort that someone might experience when asked to perform mathematically and causes low self-esteem, feelings of tension, helplessness, mental disorganization, stress, and worry.

Teachers' sense of self-efficacy is a construct derived from Bandura's (1997) theory of self-efficacy. This theoretical framework is a two-dimensional construct in which the generalized behavior of an individual is based on: (a) a belief about action and outcome, particularly a factor that relates to a teacher's sense of teaching efficacy, or belief that a teacher's ability to bring about change is limited to factors external to the teacher such as parental influences, home environment, and family background, and (b) a personal belief about one's own ability to cope with a task, such as a factor that relates to a teacher's sense of personal teaching efficacy, or belief that they have the skills to bring about student learning. When investigating teacher efficacy Zee and Koomen, (2016) found that highly efficacious teachers use a variety of instructional strategies such as inquiry based instruction, student-centered teaching strategies, are willing to use manipulatives, strive to implement new strategies, and share the control of learning with their students. Those teachers with a low sense of efficacy are more likely to use teacher-directed strategies such as lecture, straight text reading, and very little, if any, problem-solving strategies in the classroom.

Improving the quality of mathematics instruction will help reduce preservice teachers' mathematics anxiety (Beilock & Maloney, 2015). By being aware of preservice teachers' mathematics anxiety and instructional methods that contribute to it or help to reduce it, teacher education programs can better identify and support preservice teachers' needs and provide opportunities to raise mathematics self-efficacy and mathematics teachers' efficacy (Taylor & Fraser, 2013). Elementary preservice teachers' participation in a mathematics methods course corresponded to significant increases in mathematics teaching efficacy (Zee & Koomen, 2016). Several studies investigating teacher efficacy beliefs indicate that beliefs may account for individual differences in teachers' effectiveness (Suarez-Pelliconi, Nunez-Pena, & Colome, 2014) and that teacher effectiveness is associated with mathematics anxiety (Gresham, 2017). It does appear that mathematics anxiety may be linked to teacher efficacy. Therefore, the purpose of this research study was to investigate the following research questions: (a) What is the relationship between elementary preservice teachers' mathematics anxiety and mathematics teacher efficacy?, and (b) What are elementary preservice teachers' perceptions toward their mathematical skills and abilities to teach elementary mathematics effectively?

Methodology

Participants and Setting

Three hundred forty-seven elementary education preservice teachers participated in the study. Of the 347 participants, 332 were female, and 15 were male. All were working toward a K-6 endorsement in elementary education from the state and had completed at least two university mathematics courses and one elementary mathematics content course. The elementary mathematics methods course placed extensive emphasis upon the reform vision of the National Council of Teachers Mathematics (NCTM, 2000) including (a) communication, (b) problem-solving, (c) connections, (d) representation, and (e) reasoning and proof. The course was designed to help preservice teachers develop an understanding of mathematics, mathematics pedagogy, and children's mathematical development; and to cultivate a positive disposition toward teaching mathematics, and lessen preservice teachers' mathematics anxiety. The design included professional readings, group discussions, writing about the philosophical underpinnings of different approaches to teaching and learning with a focus on the role of the teacher and students, exploration through hands-on approaches and strategies, and manipulatives use. They were also required to design and implement activities and teach a minimum of four investigative lessons using manipulatives with children during their 12-week internship experience in the schools. Both the instructor and preservice teachers modeled investigative lessons focusing on problem-solving strategies, hands-on opportunities, manipulative use, integration of children's literature, and technology. The instructor facilitated discourse with preservice teachers after each lesson's presentation.

Data Collection and Analysis

The Mathematics Anxiety Rating Scale (MARS) developed by Richardson and Suinn (1972), the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), (Enochs, Smith, & Huinker, 2000), and oral interviews were used. Of the 347 preservice teachers, 40 were selected for 40-45 minute individual interviews (20 with the highest levels of mathematics anxiety and 20 with the lowest levels mathematics anxiety according to the MARS). The interview questions used in this study were adapted from Swars, Daane, and Giesen's (2006) interview protocol. Interviews were audiotaped, then coded for themes.

Findings

The results revealed a moderate negative relationship between mathematics anxiety and mathematics teacher efficacy among preservice teachers on the MTEBI (see Table 1 and 2).

Table 1

Means and Standard Deviations for Mathematics Teaching Efficacy and Anxiety Scores

Scale	Mean	Standard Deviation
MARS	231.69	65.45
Personal Mathematics Teaching Efficacy Subscale	52.81	8.77
Mathematics Teaching Outcome Expectancy Subscale	33.57	3.41
MTEBI Subscales Combine	84.44	8.12

Table 2

*Pearson Product Moment Correlations between Mathematics Teacher Efficacy and Anxiety Scores**

MTEBI	MARS Total Score
Personal Mathematics Teaching Efficacy Subscale	-.481**
Mathematics Teaching Outcome Expectancy Subscale	-.022
MTEBI Subscales Combined	-.456**

*n = 347
**Correlation is significant at the .05 level (2-tailed)

Scores indicated that the preservice teachers with higher levels of mathematics anxiety had lower mathematics teacher efficacy and the preservice teachers with low levels of mathematics anxiety had higher mathematics teacher efficacy. Analysis of the relationship between the MARS and the Mathematics Outcome Expectancy subscale revealed no relationship. Preservice teachers' levels of mathematics anxiety and their beliefs that effective teaching can bring about student learning regardless of external factors indicated no significant relationship.

Interview data showed the emergence of four themes. These themes related preservice teachers' perceptions toward their effectiveness in teaching mathematics to elementary students. These themes included attitudes towards mathematics, mathematics teaching practices, foundation of efficacy beliefs, and the description and understanding of mathematics. During the interviews, preservice teachers with the highest levels of mathematics anxiety expressed negative attitudes towards mathematics. All 20 indicated they did not like the subject, really struggled with the content, and expressed negativity towards mathematics. In contrast, during preservice teachers' interviews those who had low levels of mathematics anxiety indicated positive attitudes towards mathematics. Eighteen of 20 indicated they enjoyed mathematics and expressed a strong desire to learn as much as they could to help those who did not like the subject. All stated that the use of manipulatives, using problem-solving situations, and real-world experiences in the mathematics classroom was very important for teaching and motivating students to learn mathematics. The use of manipulatives, incorporating problem-solving situations, and real-world experiences were a requirement for their investigative teaching experience lessons in their internship. They expressed the need for including these strategies for successful learning, indicating they would have used them anyway, even if not required.

Thirty-eight of 40 preservice teachers felt comfortable with and enjoyed teaching with manipulatives. Although 29 of 40 indicated they were unsure in the beginning, they felt that as time passed and they became more comfortable in front of their students and with teaching, that using manipulatives became an essential learning tool in the classroom. They expressed much stress learning how to use manipulatives and use them to teach students. All 40 posited no involvement with manipulative use during mathematics learning when they were in elementary school. Twenty-four of the 40 preservice teachers interviewed revealed that they believed they could teach mathematics effectively. Those that indicated they were unsure had high levels of mathematics anxiety. They also stated struggles with the pressure of going to class and internship, developing their own lesson plans, having to teach all day, dealing with personal issues, and understanding the mathematical content. Some preservice teachers commented about difficulties to learn mathematics, lack of help from their parents who had little knowledge of the mathematical content, and the challenge to meet their weaknesses in mathematics throughout their own school years. Others expressed that they could and did teach mathematics effectively because of their understanding of how hard it was to grasp and learn mathematics. They expressed feelings of frustration when doing mathematics, embarrassment, feeling "stupid" in front of others, overwhelmed, and anxious; all thoughts held by many with high levels of mathematics anxiety. In contrast, those preservice teachers with lower levels of mathematics anxiety found that being challenged in mathematics was fun; thus allowing them to create an enthusiastic learning environment and positive attitude toward mathematics.

Discussions and Conclusions

The surveys showed that the preservice teachers who were highly efficacious had lower mathematics anxiety levels. Those preservice teachers with higher mathematics anxiety levels had lower levels of teacher efficacy. Several of those with high levels of mathematics anxiety expressed some efficaciousness. However, they continued to doubt their ability to teach effectively due to their mathematics anxiety. The results of the study suggest that mathematics anxiety does have a negative relationship with a preservice teacher's belief in his or her own skills and abilities to be an effective teacher. The results from the study supported others that described mathematics anxiety as a cause of the lack of preservice teachers' confidence in educational activities (Gresham, 2017; Beilock & Maloney, 2015; Swars, Daane, & Giesen, 2006). The findings of this study also are consistent with Nisbet (2015), Taylor and Fraser (2013), and Swars, Daane, and Geisen (2006) studies. They found that highly efficacious preservice teachers had lower mathematics anxiety levels and those that had lower efficacious beliefs had higher mathematics anxiety levels. This study's findings contribute to the research of preservice teachers' mathematics anxiety and mathematics teacher efficacy by having a much larger sample size. However, the study also differs because preservice teachers were required to teach an investigative lesson using manipulatives to classmates in the methods course. They were involved in in both large and small group weekly class discussions on problem-solving strategies, hands-on opportunities, manipulative use, integration of children's literature, and technology use within their internship experience. They were also responsible for the preparing and teaching a minimum of four lessons within their internship that included manipulatives, problem-solving strategies, hands-on opportunities, and children's literature.

The purpose of this study was to provide a more in-depth look at preservice teachers' mathematics anxiety and mathematics teacher efficacy. This study indicated no relationship between preservice teachers' mathematics anxiety and their beliefs that effective teaching can bring about student learning of mathematics regardless of external factors. The research by Stoehr, (2012) indicated that preservice teachers often exhibit an unrealistic attitude toward their abilities to overcome negative external influences. Stoehr expressed that teaching outcome expectancy beliefs may actually be difficult to measure due to the number of variables involved. Therefore, further research regarding this factor should be conducted. Although the results of this study seem to suggest that mathematics anxiety has a negative relationship with mathematics teaching efficacy, the interviews indicated that preservice teachers with the highest levels of mathematics anxiety were somewhat efficacious about their skills and abilities to teach mathematics effectively. The interview results suggest that preservice teachers' attitudes toward mathematics play a crucial role in their beliefs to teach mathematics effectively. Data showed that preservice teachers with negative attitudes toward mathematics had the highest levels of mathematics anxiety, while those with lower mathematics anxiety levels had more positive mathematics experiences. Looking at the interview data from the preservice teachers, this result is not surprising. Therefore, teacher education programs should be concerned with helping preservice teachers make connections between mathematics and the quality of mathematics education needed for everyone. Expecting preservice teachers with both high and low mathematics anxiety to have the same confidence levels to teach seems unrealistic. Even if the anxiety is low, they are not free from it. NCTM (2014) indicated that all teachers should focus upon mathematics content and processes that are worthy of students' time and attention.

Regardless of their mathematics anxiety levels, all preservice teachers interviewed indicated the importance of using manipulatives, relating mathematics experiences to the student's real-world environment, and motivating them. It is important for the mathematics curriculum to provide experiences that help students see the importance of mathematics by modeling a variety of problem solving strategies and techniques. Previous studies indicated that instruction focused on traditional approaches to teaching mathematics create higher levels of mathematics anxiety than those classrooms whose teachers employ more non-traditional practices (Gresham, 2017; Sloan, 2010). As preservice teachers discussed their teaching practices all expressed a sense of efficaciousness toward using authentic situations that focused on meaningful experiences.

Conclusion

Researchers have long regarded teachers as one of the most significant factors in improving children's learning and educational outcomes. Teachers' mathematics anxiety and efficacy beliefs toward mathematics can powerfully influence how mathematics is taught. It also affects how children will ultimately come to view mathematics (Taylor & Fraser, 2017). Many teachers in the United States, however, lack the necessary skills and competence to teach mathematics in a way that promotes understanding and helps children develop positive mathematical attitudes and efficacy beliefs (Zee & Koomen, 2016). There are previous studies that have indicated that mathematics methods courses have been effective in reducing mathematics anxiety. The results of this study are consistent with previous research between teacher efficacy and classroom instructional strategies (Swars, Daane, & Giesen, 2006). Central to further research is a need to determine how mathematics anxiety and mathematics teaching efficacy beliefs influence teaching practices and subsequent student achievement. Therefore, if teacher education programs hope to influence the development of effective instructional practices, an important part of the teacher education program should focus on the development of mathematics teacher beliefs and the reduction of mathematics anxiety in preservice teachers.

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ELEMENTARY PRESERVICE TEACHERS' PERCEIVED CONFIDENCE AND READINESS FOR TEACHING MATHEMATICS

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This study examined the self-efficacy of preservice elementary teachers enrolled in a three-course, three-semester mathematics sequence. The three-course mathematics sequence was taught from a constructivist approach and emphasized a socioconstructivist learning environment in which the preservice teachers were challenged to construct their own meaning of mathematics. Results showed that the preservice teachers' confidence and readiness levels for teaching elementary school mathematics increased by the end of their enrollment in the three-course mathematics sequence. However, the preservice teachers' self-efficacy with regards to their beliefs that effective teaching can affect student achievement remained the same, suggesting further research in this area.

Efforts in mathematics education reform (NCTM, 2000) advocate ideologies consistent with constructivist theory for teaching and learning mathematics. Constructivist teaching provides opportunities for learning mathematics via participating in meaningful discourse about mathematics, constructing one's own knowledge and understanding of mathematics, and reflecting on processes of solving mathematics problems (Steffe & D'Ambrosio, 1995; von Glasersfeld, 1983). In light of these reform efforts, current teacher education programs have emphasized a constructivist learning environment.

The purpose of this study was to investigate the extent to which a three-course mathematics sequence taught from a constructivist approach would increase elementary preservice teachers' perceived confidence and readiness for teaching mathematics. Research (Hart, 2002) has shown that teacher education programs have the propensity to shape teachers' perceptions of what mathematics is important and appropriate to teach and how that mathematics should be taught. Further, several research studies (Swars, Hart, Smith, Smith, & Tolar, 2007; Hart, 2002) have found that preservice teachers' self-efficacy beliefs affect how they perceive and think about the many ideologies with which they are faced in their teacher preparation programs. Some scholars have argued that teachers' self-efficacy beliefs develop over time and are resistant to change (Pajares, 1992). Therefore, in order to effect change with regards to self-efficacy beliefs, teacher education programs must provide opportunities for preservice teachers to examine and reflect on their beliefs.

According to Hart (2002) and Swars et al. (2007), research studies on teachers' beliefs are usually examined in a single methods course. Unfortunately, "if mathematics content is taught by lecture and the methods courses use a constructivist environment, the experience is diluted, and the chance for change is significantly decreased" (Hart, 2002, p. 5). By contrast, this research examined preservice teachers' self-efficacy and change in self-efficacy beliefs over a period of three semesters in which the preservice teachers participated in a sequence of three required content mathematics courses, each taught using a socioconstructivist learning environment.

Theoretical Framework

The influence of preservice teachers' self-efficacy beliefs on their conceptions of mathematics teaching and learning is well documented in the literature. Although several research studies in mathematics education have been conducted on teachers' beliefs, the authors specifically used Bandura's (1977, 1986) work on self-efficacy to frame this study. Bandura (1977) defined self-efficacy as individuals' viewpoints, perceptions, or judgments of their capabilities to succeed. Self-efficacy beliefs influence how persons think, motivate themselves, and ultimately, behave (Bandura, 1993). With regards to preservice teachers, self-efficacy beliefs influence how they think about mathematics, how they think about teaching mathematics, and how they respond to the process of learning to teach mathematics.

Bandura's (1977) theoretical perspective on self-efficacy emphasizes two constructs: *personal teaching efficacy* and *teaching outcome expectancy*. The *personal teaching efficacy* can be defined as a teacher's belief in their own knowledge, skills, and abilities to be an effective teacher. The *teaching outcome expectancy* can be defined as a teacher's belief in the notion that student learning hinges on effective teaching, and moreover, effective teaching can illicit student achievement and success regardless of external factors affecting the student.

Teacher education programs consistent with past reform efforts (NCTM, 2000) in mathematics education emphasize programs in which preservice teachers construct their own knowledge and understanding of mathematics and focus on created socially-constructed learning opportunities. However, preservice teachers usually enter their teacher preparation programs with ideologies and self-efficacy beliefs that are inconsistent with this philosophy for teaching and learning. Further, preservice teachers' self-efficacy beliefs determine how much effort they will initiate, expend, and sustain in the face of aversive experiences (Bandura, 1977), such as participating in a mathematics learning environment that is paradoxical in relation to their prior schooling experiences with mathematics. This research particularly sought to answer: (a) to what extent does participation in a three-course mathematics sequence taught from a constructivist approach impact preservice teachers' confidence and readiness for teaching elementary school mathematics? and (b) to what extent does participation in a three-course mathematics sequence taught from a constructivist approach increase preservice teachers' personal mathematics teaching efficacy and their mathematics teaching outcome expectancy?

Methodology

Twenty-seven elementary preservice teachers participated in a three-course mathematics sequence as part of the requirements necessary to complete their teacher certification. Twenty-five of the preservice teachers were females. Of these 25 females, 21 were Caucasian, one was Bosnian, one was Asian American, one was African American, and one was Native American. The remaining two preservice teachers were Caucasian males.

The three-course mathematics sequence was taught over the course of three independent semesters. Each course in the sequence focused on a particular set of mathematics topics. The focus of the first course was number sense, whole number and integer operations, and algebraic thinking. The second course emphasized the study of geometric concepts with a focus on both two- and three-dimensional shapes, and the third course emphasized the study of rational numbers, data analysis, and probability.

One limitation of the study is that the first author was the instructor of record for each course taught in the three-course sequence. Although both authors developed materials for the courses, the authors recognize that there is inherent bias due to the first author's teaching style and personality.

The intent of each course was to emphasize a social-constructivist classroom environment. As such, each course emphasized teaching and learning consistent with the philosophy of reform in mathematics education (i.e., NCTM, 2000). Further, the preservice teachers were challenged to investigate mathematics problems, find their own solutions, and discuss and justify those solutions. Mathematics instruction in all three courses emphasized conceptual understanding, and as a consequence, the preservice teachers were challenged to determine *why* specific algorithms and procedures worked for solving particular mathematics problems. Also, the preservice teachers connected concrete and theoretical mathematical models and used manipulatives, problem solving, and reflection to make sense of concepts and to construct their own meaning of mathematics.

Procedure and Data Analysis

At the beginning of the first course in the three-course sequence, preservice teachers completed the *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBI) (Enochs, Smith, & Huinker, 2000) and then again at the end of the third course in the mathematics sequence. The MTEBI consists of 21 items and is a Likert-scale instrument that has five response categories: strongly agree, agree, uncertain, disagree, and strongly disagree. Higher scores on the MTEBI indicate a greater teaching efficacy with lower scores indicating a lower teaching efficacy.

Thirteen of the MTEBI items are classified as the *Personal Mathematics Teaching Efficacy* (PMTE) subscale, and eight are classified as the *Mathematics Teaching Outcome Expectancy* (MTOE) subscale (Enochs, Smith, & Huinker, 2000). The PMTE subscale addresses preservice teachers' self-efficacy about their capabilities—specifically, their own knowledge and skills—to become effective mathematics teachers. The PMTE consists of the following thirteen items.

Table 1

Personal Mathematics Teaching Efficacy

Item 2	I will continually find better ways to teach mathematics.
Item 3*	Even if I try very hard, I will not teach mathematics as well as I will most subjects.
Item 5	I know how to teach mathematics concepts effectively.
Item 6*	I will not be very effective in monitoring mathematics activities.
Item 8*	I will generally teach mathematics ineffectively.
Item 11	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.
Item 15*	I will find it difficult to use manipulatives to explain to students why mathematics works.
Item 16	I will typically be able to answer students' questions.
Item 17*	I wonder if I will have the necessary skills to teach mathematics.
Item 18*	Given a choice, I will not invite the principal to evaluate my mathematics teaching.
Item 19*	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.
Item 20	When teaching mathematics, I will usually welcome student questions.
Item 21*	I do not know what to do to turn students on to mathematics.

**Indicates negatively stated item*

The PMTE was scored as follows: strongly agree = 5; agree = 4; undecided = 3; disagree = 2; strongly disagree = 1. Reverse coding was used to score *negatively* stated items. For example, items such as item 3, *Even if I try very hard, I will not teach mathematics as well as I will most subjects*, were categorized as negatively stated items. Therefore, they were scored using reverse coding – strongly agree = 1; agree = 2; undecided = 3; disagree = 4; strongly disagree = 5.

The MTOE subscale addresses preservice teachers' self-efficacy about effective teaching augmenting students' mathematics achievement regardless of external factors that may influence student learning. The MTOE consists of the following eight items.

Table 2

Mathematics Teaching Outcome Expectancy

Item 1	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.
Item 4	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.
Item 7	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.
Item 9	The inadequacy of a student's mathematics background can be overcome by good teaching.
Item 10	When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.
Item 12	The teacher is generally responsible for the achievement of students in mathematics.
Item 13	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.
Item 14	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.

Although the MTEBI is widely used, and researchers have found it to be both reliable and valid, there are some documented limitations. According to Kieftenbeld, Natesan, and Eddy (2011), the wording and placement of items in the MTEBI might be sources of local dependence, which occurs when some items are near duplicates. Further, items on the PMTE subscale that are negatively worded may lead to erroneous analyses (Kieftenbeld, Natesan, & Eddy, 2011).

To analyze the data appropriately, the authors used descriptive statistics to analyze the distribution of the preservice teachers' responses on the MTEBI. Since this research studied a small cohort (n = 27) of preservice teachers, descriptive statistics were used as a way to examine propensities across the group with respect to the two constructs – *personal mathematics teaching efficacy* and *mathematics teaching outcome expectancy*. The authors specifically used the PMTE and the MTOE subscales to independently examine two categories of teaching efficacy beliefs.

Paired-samples t-tests were used to analyze the preservice teachers' mean score differences from pretest to posttest and to compare mean score differences within the group of preservice teachers. Specifically, the authors tested the null hypothesis that there were no differences in the preservice teachers' levels of self-efficacy at the end of the three-course mathematics sequence. Since the authors were testing the null hypothesis that the two mean scores from pretest to posttest were equal, a two-tailed test was used.

Findings

To determine whether participation in a three-course mathematics sequence taught from a constructivist approach increased the preservice teachers' personal mathematics teaching efficacy and their mathematics teaching outcome expectancy, the authors used the work of Bandura (1977, 1986, 1993) and independently examined the two constructs from the MTEBI: *Personal Mathematics Teaching Efficacy* (PMTE) and *Mathematics Teaching Outcome Expectancy* (MTOE).

The authors analyzed the mean score differences in self-efficacy and tested the null hypothesis that there were no differences in the mean scores from pretest to posttest. As shown in Table 3, the authors found that for the PMTE subscale, self-efficacy levels were significantly different at the end of the three-course mathematics sequence than at the beginning. This suggests that the preservice teachers' self-efficacy with regards to their perceived capabilities for teaching mathematics effectively increased by the end of their enrollment in the three-course mathematics sequence.

Table 3

Paired-Samples t-test Efficacy Results

Assessment	*Mean*	*Standard Deviation*	*t–value*
PMTE			
Pre-Test	3.915	0.486	
Post-Test	4.051	0.477	**2.606****
MTOE			
Pre-Test	3.708	0.370	
Post-Test	3.741	0.417	0.462
N = 27	***df = 26***	***Two-tailed***	*****p < 0.01***

However, for the MTOE subscale, the authors did not find a significant difference in the self-efficacy levels from pretest to posttest. It is important to note here that along with their enrollment in the third mathematics content course, the preservice teachers were simultaneously enrolled in a *block* of methods courses and field placements. The methods courses and field placements included content areas other than mathematics. It is the authors' hypothesis that the preservice teachers' teaching outcome expectancy beliefs were the same at the end of the three-course mathematics sequence as they were at the beginning because of this participation in multiple field placements and methods courses.

Some research studies (Swars et al., 2007) have found that preservice teachers' mathematics teaching outcome expectancy (MTOE) beliefs tend to remain the same during student teaching, while other studies (Hoy & Woolfolk, 1990) have found that preservice teachers' self-efficacy beliefs with regards to their MTOE decline significantly during student teaching. It is the authors' contention that during student teaching or even during participation in multiple field placements, preservice teachers may become overwhelmed with being immersed in teaching and learning environments that may be inconsistent with the constructivist teaching and learning environments emphasized in their teacher preparation programs. Consequently, preservice teachers may not feel empowered or may not feel as though they are able to control whether students perform well or achieve in mathematics.

Conclusions

This research investigated preservice teachers' perceived confidence and readiness for teaching elementary school mathematics. The authors sought to determine whether and to what extent a three-course mathematics sequence taught from a constructivist perspective would increase preservice teachers' beliefs with regards to their personal mathematics teaching self-efficacy and their mathematics teaching outcome expectancy self-efficacy.

With regards to preservice teachers' confidence and readiness for teaching mathematics and their personal mathematics teaching efficacy, the results of the study show that these self-efficacy beliefs increased by the end of their participation in the three-course mathematics sequence. Although several scholars (Pajares, 1992) have cautioned that preservice teachers' beliefs develop over time, are influenced by prior experiences as students themselves, and are resistant to change, this study revealed that preservice teachers' learning mathematics from a constructivist approach and participating in a socioconstructivist mathematics learning environment can change their beliefs. However, since this study only examined preservice teachers' beliefs over the course of three semesters, it is unclear whether this change in beliefs is *lasting* change. In other words, it cannot be predicted whether this change in beliefs will continue into practice as the preservice teachers become inservice teachers.

Since the results of the study show that the three-course mathematics sequence had no impact on changing the preservice teachers' mathematics teaching outcome expectancy beliefs, it is important to conduct further research in this area. Perhaps the collection of qualitative data such as participant interviews could help determine why self-efficacy beliefs in this area are more resistant to change than preservice teachers' perceived confidence in their skills and abilities to effectively teach mathematics.

Although this study was limited to 27 preservice teachers and cannot be generalized to every cohort of preservice teachers who participate in the aforementioned three-course mathematics sequence, this study has added to the existing and growing body of literature on teachers' beliefs. In particular, this study has provided insight into the essential components needed in teacher education programs to effect change in preservice teachers' self-efficacy beliefs. Moreover, unlike most studies on teachers' beliefs that examine the impact on beliefs of preservice teachers' participating in methods courses, this study specifically investigated the impact on beliefs of preservice teachers' participating in content mathematics courses. In terms of teacher education programs' changing preservice teachers' beliefs, this research study provides evidence that although change is difficult, it can happen.

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"AWFUL" AND "FUN": TEACHERS' MIXED PERCEPTIONS OF CONTENT-FOCUSED PD

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A clear connection exists between the need for content-focused professional development and student learning; to teach conceptually, teachers must have a deep understanding of the content and how students learn that content. This study examines how teachers' who did and did not choose to participate in year-long content-focused training (a) perceived the importance of the content-focused training and (b) the factors they considered when deciding whether to attend or not attend the training sessions. Overall, math circles were perceived to increase teachers' mathematical knowledge for teaching, confidence in math ability, and reasoning strategies needed to teach math through problem solving.

Rationale

Teachers with the greatest need for improvement in mathematics content knowledge are the least likely to take sustained, content-focused professional development [PD] (Desimone, Smith, & Ueno, 2006). This becomes concerning as student learning is more likely to improve if the professional learning that teachers participate in increases their understanding of the content they teach, how students learn the content, and how to represent and communicate that content in a significant way (Cohen & Hill, 2000).

To represent the knowledge needed to teach math, Hill, Rowan, and Ball (2005) defined mathematical knowledge for teaching [MKT] as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). MKT has been further conceptualized to include two domains. First, subject matter knowledge represents knowledge that is purely mathematical in nature (Ball, Thames, & Phelps, 2008). Second, pedagogical content knowledge represents the content knowledge most connected to instruction (Ball et al., 2008).

A common means by which teachers MKT may be positively affected is PD (Hill & Ball, 2009). To understand effective PD, Birman and colleagues (2000) identified six features of PD: form, duration, participation, content focus, active learning, and coherence (Birman, Desimone, Porter, & Garet, 2000). They identified a longer duration as providing greater opportunities to develop content knowledge and the focus of the PD on content knowledge being directly related to teachers' increases in knowledge (Birman et al., 2000).

Given the importance of teachers' MKT on students understanding of mathematics, our study is designed to further examine how middle school math teachers who did and did not choose to participate in year-long content focused PD (a) perceived the importance of the content-focused training and (b) the factors they considered when deciding to attend or not attend the training sessions.

Methods

Context of the Professional Development

The study included the participation of middle school math teachers in content-focused PD. The PD was led by three mathematics faculty members and one secondary mathematics education faculty. The PD was completed in two parts: (1) five PD sessions were held throughout the academic school year; referenced as year-long PD, and (2) nine PD sessions were held across two weeks during the summer; referenced as summer PD.

The content of the year-long PD was framed by math circles. Math circles allow teachers to learn to understand a problem, experiment with various problem-solving techniques, articulate and test those conjectures, and present possible solutions to their peers. The content of the summer PD was a combination of math circles and Connected Mathematics Project 3 (CMP3) curriculum training. The curriculum training included grade-level sessions that provided opportunities for teachers to work and discuss specific units within the curriculum. For teachers to receive the CMP3 curriculum, participation in the full nine days of summer PD was required.

Participants

The participants included 23 middle school teachers from three counties in the Southeastern United States. All 23 attending the summer PD were also invited to the year-long PD; 18 of the 23 teachers attended at least one of the five sessions within the year-long PD. The 23 teachers were from ten partner schools, eight of the ten schools were identified as high needs by percentage of students not meeting proficiency levels in Mathematics on the ACT Aspire Test, 2013-2014. On average, 77.06% of the students from the 10 schools were not meeting the content standards for math.

Of the 23 teachers, 12 teachers had 0-5 years of teaching experience and 11 teachers had more than 6 years of teaching experience. For teacher grade level and certification, see Table 1. The category of "Other" represents those teachers teaching more than one grade-level.

Table 1

Teachers' grade level and certification

	Teacher Certification					
Grade Level	Elementary Grades	Middle Grades	Secondary Grades	Elementary & Special Edu	Middle & Special Edu	Total
6	6	1	0	1	0	8
7	0	2	0	0	2	4
8	0	4	2	0	0	6
Other	1	1	2	0	1	5
Total	7	8	4	1	3	23

Data Collection and Analysis

Data sources included two focus group interviews, an end of summer-PD survey, distance from each participant's school to the PD site, and pre- and post- content assessments. Six different participants were randomly sampled for each focus group interview, two of whom did not attend during the year-long PD and four of whom did attend during the year-long PD.

Data from the focus groups and open-ended survey responses were independently open-coded by the first and second authors. Following the initial coding, themes were independently developed around topics related to each research question. Themes were then compared and consensus was reached. Data from pre- and post- content assessments were analyzed using a paired samples *t*-test to examine growth in teachers' content knowledge connected to participation in the PD. Finally, quantitative data from the end of PD survey was analyzed using bivariate correlation to examine the impact physical distance may have had on teachers' willingness to participate in the PD.

Results

Perceived Importance of Content-Focused PD

Opportunities for learning. Most generally, participants spoke of learning taking place every time they attended a math circle meeting. Such learning included an increased confidence in their math ability and a learned ability to view problems from multiple perspectives. One participant elaborated in saying that problems were approached in ways he/she would have never considered while another participant stated that they were learning to look at problems from the perspective of a mathematician.

Next, the teachers discussed how the math circles provided opportunities for them to learn to think in the same manner the CMP3 curriculum requires of their students. One teacher stated, "I like being able to be in the same seat as my student versus already knowing the math and kind of getting caught up with that struggle" (Participant 4, Focus Group 1). A last manner in which learning was commonly discussed related to supplementing the content knowledge of the teachers with limited math backgrounds; the learning opportunities provided throughout the math circles were described as supplementary math courses or classes.

The quantitative data provided some evidence that teachers did experience an increase in content knowledge connected to their participation in the PD. Results of paired samples *t*-test indicated a positive change that was approaching significance ($t = 1.815$; $p = 0.084$), and examination of Cohen's d revealed a medium-size effect ($d = .387$) for the teachers' increase in content knowledge from the beginning of their participation in the PD to the end. Failure to achieve statistical significance was likely due to the limited sample size.

Few participants were more critical of the learning opportunities, most commonly that the content was too difficult. When asked, one participant stated, "The content was by far the least effective aspect of math circles. The presenters or professors seems to assume that we are all strong in math. Many of us are not" (Survey Response #8). An additional participant felt they were in the role of student learner too often.

Application to teaching. Participants had mixed perceptions of if and how the math circles may impact their teaching. First, many participants discussed how increasing their own content knowledge would have a direct effect on their teaching. For example, one participant stated,

> I think to, the more we kind of stretch our brains, that gets delivered right back to the kids because now I have other ways to present content to them or show them multiple ways to solve problems that maybe I wouldn't have even thought about had it not been for being exposed to math circles. (Participant 2, Focus Group 2)

Other participants shared a similar sentiment in that they appreciated opportunities to try problems above their grade level and the confidence that brought to their own teaching.

Next, many participants felt that the problems within the math circles have evolved to be more appropriate for their students and applicable to their classrooms, including discussions around how to implement the tasks with students. Numerous participants also described how comfortable the mathematics professors made them feel when struggling with a challenging problem and that the professors were teaching them how to react when their own students' struggles. For example, one participant stated, "You're bringing that back to your kids because you realize, oh, I remember last Wednesday what it was like to struggle with that problem so I need to take deep breathes, sit back and let's look at it another way" (Participant 2, Focus Group 2).

Those more critical of how the math circles might apply to teaching did not enjoy engaging with problems above their grade level, specifically the sixth grade teachers. Few teachers felt the math circles would be more beneficial if they were separated by grade-level. Participants also felt that the math circles lacked connections to the CMP3 curriculum and/or how they would teach that concept. Lastly, multiple participants expressed a desire to have the correct solutions provided following math circles for future reference.

Community building. Teachers' perceptions of community building included little variation. First, teachers often discussed the benefit of collaborating when solving the math problems. Likewise, when asked what was the most effective aspects of the math circles, multiple teachers discussed opportunities to work with other teachers. The discussions around collaboration also included receiving support for teaching. For example, consider how one teacher discussed the support received from the collaboration across the meetings:

> …you just started building relationships with teachers from magnet schools, from city schools, from county schools just to determine how did you approach this with your kids. How did this work? And it will kind of encourage you to do it in your classroom and then you begin to share resources and ideas and share lessons and it just makes it, makes it fun and it makes it easier (Participant 4, Focus Group 1).

The relationships developed within the math circles extended support for teaching throughout the school year and beyond math circle meetings. One participant discussed the importance of maintaining communication with the math circle participants because they have received similar training and that they would in turn have a similar perspective.

Factors Considered when Attending or not Attending PD

Time and distance. Participants were split in their preference for holding math circle PD during the school day versus after school hours. For many, mandated school schedules and testing were consistently listed as reasons for not being able to attend. Likewise, teachers mentioned the difficulty of being out of the classroom for multiple days. Some teachers mentioned the alternative of having math circle meetings after school while others discussed conflicts due to after school obligations, such as coaching or their own children.

Distance was also a factor we considered when examining teachers' participation in math circles. Analysis of quantitative data revealed a significant negative correlation ($r = -0.379$, $p = 0.47$) between the number of miles teachers had to travel in order to attend the PD and the teachers' overall amount of participation. Five of the six teachers that participated fully in both the year-long PD and the summer PD needed to travel less than 10 miles from their school to the PD. Conversely, none of the teachers mentioned distance to the PD as an issue within any of the qualitative data collection.

Administrative support. Participants were also split in the support they received from their administrators. The participants mentioned that many of their administrators lack a mathematics background and because of this, they don't feel their administrators understand the type of PD they need to implement effective math instruction or even what effective math instruction looks like. Many of the participants recommended providing additional training for administrators around math education.

Discussion

The learning opportunities within math circles support teachers in their development of both content knowledge and pedagogical content knowledge. Teachers' development of content knowledge was evident through their discussions of learning to examine problems in multiple ways, in their collaboration around solving challenging problems, and in the medium effect size of content knowledge across the pre-and post-assessment. As a result of increased content knowledge, we can expect teachers to be better prepared to explain the reasoning underlying a procedure, determine if a mathematical argument is valid, and to select the most appropriate mathematical representations within their own classrooms (Ball et al., 2008). In contrast, few teachers felt that engaging in problems beyond their specific grade-level was a least effective aspect of the math circles, many of which were sixth grade teachers.

The teachers also discussed their learned teaching strategies, or improved pedagogical content knowledge, as result of attending math circles. Many of the participants perceived that a combination of the implemented math tasks, getting to see various manners in which others thought about that task, and discussions around how students would solve the task would directly benefit their classroom instruction. In alignment with literature on effective PD, both engaging in the mathematics and thinking about how students would engage with the math proved effective (Garet, Porter, Desimone, Birman, & Yoon, 2001). Opportunities to struggle with math and reflect on how the professors responded to these struggles were also discussed as strategies the participants would be able to use in supporting their own students. In alignment with recommendations for supporting productive struggle in learning, the participants reported that their approaches and thinking around a specific math problem were valued over a correct solution (National Council of Teachers of Mathematics, 2014).

The math circles were also effective in developing a community of educators. In the teachers' discussions, it became evident that relationships extended beyond the math circle meetings. Also, opportunities to collaborate ranked highest when surveyed about various aspects of math circles. Although some literature on effective PD discusses the importance of having all teachers within a grade-level attend a PD (Birman et al., 2000), value was shown in collaborating with those who may teach in a different setting than themselves.

When determining the factors teachers considered when attending math circles, time, and administrative support were discussed most commonly. Although literature reports the effectiveness of PD with a great number of contact hours (Birman et al., 2000; Clarke & Hollingsworth, 2001), identifying a common time proves difficult. Administrators were also reported to hold varying views of content-focused PD in allowing or not allowing teachers to attend; some teachers hypothesized these decisions may have been affected by their administrators' lack of a math background.

Conclusion and Implications

The perceived importance of the content-focused PD was evident through opportunities for learning mathematics content, an increased pedagogical content knowledge, and the addition of a support community. Factors considered when deciding to attend or not attend the content-focused PD included time, distance, and administrative support. Overall, math circles were perceived to increase teachers' mathematical knowledge for teaching, confidence in math ability, and reasoning strategies needed to teach math through problem solving. Likewise, collaboration within math circles offered teachers support beyond the PD needed to teach using reform-oriented math curriculum.

A first implication of the study is a need for explicit conversations around the importance and affordances of increased math knowledge. In attending the same training, some teachers saw the benefit in engaging with challenging math beyond their grade level while others were dissatisfied and expressed a desire to only engage in math circles within their grade-level.

A second implication is a need for conversations or training with middle school administrators. As discussed, the teachers participating within this study were from low-performing schools. It is possible the administrators valued the teachers being present for instruction over allowing teachers attending content-focused PD. Given not all administrators have a math background, training and/or conversations with administrators are needed around effective math instruction and the knowledge need to be an effective math teacher. Likewise, given those teachers who need content-focused PD are the least likely to attend, administrators are key in ensuring students have access to quality math teachers and instruction (Desimone et al., 2006).

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LEARNING MATHEMATICAL PROOF
CONCEPTUAL CHALLENGE OR ESOTERIC CULTURAL PRACTICE?

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This paper addresses the question of whether the challenges of learning mathematical proof are best theorized as challenges of conceptual understanding in the Piagetian constructivist tradition or as challenges of cultural transmission in the Vygotskyan sociocultural tradition. Utilizing a genres approach for aligning learning theory with teaching practice, we review the history of thought around this question, and analyze strengths and limitations of specific proof pedagogies.

> The third part [of our book] focusses on two larger frameworks for examining proving and argumentation. The first is argumentation processes which are social processes that occur in classrooms …. The second is reasoning processes, which are internal psychological processes (Reid & Knipping, 2010, p. xiii – xiv).

As Reid and Knipping (2010) illustrate in their introduction to *Proof in Mathematics Education*, educational scholarship on proof encompasses two distinct perspectives, a sociocultural perspective concerned with acquisition of cultural practices, and an internal psychological perspective concerned with reasoning and understanding. This matches approaches to educational reform more broadly in its dual concerns for how "knowledge is personally constructed and socially mediated" (Windschitl, 2002, p. 137).

The divergent theorization of learning in psychology has been an enduring issue for education (Cobb, 1994; Lagemann, 1989; Sfard, 1998). As Alexander (2007) noted, theorists sometimes stake out a "hard theory" stance that insists on a single frame, either "giv[ing] primacy to the individual mind … (e.g., radical constructivism) [or else] locat[ing] both the source and location of knowledge …within the immediate social context and human interactions (e.g., socioculturalism)" (p. 68). Alternatively, others adopt a "soft theory" stance that seeks to "connect the cognitive and sociocultural sides of the epistemological debate" (p. 68).

This landscape of conflicting theorizations of learning reduces the utility of theory: if theorists can't even agree on what theory is suitable, how can we place much stock in the implications of theory for teaching? Kirshner (2016) introduced a new strategy for dealing with psychology's multiple theorizations of learning. This strategy eschews the soft theory approach of integrating diverse perspectives together—as diSessa, Levin, and Brown (2016) noted in a broad overview of such efforts, to “join perspectives … represents a possible far-future outcome of the present work” (introduction). Likewise, we reject the hard theory approach of selecting a single perspective as correct, rejecting all others. Instead, the *genres* approach identifies the theorizations of learning that inspire education’s efforts, drawing sharp boundaries among them so it is clear which theory is applicable to a given educational circumstance. This strategy eliminates the conflict over perspectives that divides educators, and enables genres of teaching—distinct methods each reflecting a single theoretical perspective—to be drawn with more precision than is the case when theories are integrative or when a single theory is being stretched over all educational needs. In this paper, we use the lens of genres to examine and critique the ways in which Piagetian constructivist theory and Vygotskyan sociocultural theory have been used to inform the teaching and learning of proof.

Constructivist and Sociocultural Framing of Proof

In recognition of difficulties that undergraduates have with mastery of proof it is now not uncommon to find transition-to-proof courses that attempt to orient mathematics students to proof prior to enrolment in content courses that depend on proof (e.g., Selden, Selden, & Benkhalti, 2017). However, this is a fairly recent development (Moore, 1994, is one of the earliest references), and most of the thinking about the challenges of teaching proof (at both secondary and tertiary levels) evolved from experiences in which methods of proof are taught in conjunction with specific proofs comprising a particular mathematical content area.

This history of teaching *proof* in conjunction with teaching *proofs*, together with lack of clear boundaries among psychological theories, has led to conflation of conceptual and cultural learning challenges. Clearly, these challenges are separable: the mathematician who spends months or years working to prove a theorem is not adrift with respect to the practices of proof, but rather struggles with the conceptual complexity of the problem at hand. Conversely, the literature abounds with reports of students’ disorientation to the nature of proof. For instance, Edwards and Ward (2004) found that undergraduate mathematics majors had difficulty orienting themselves to “the *role* that mathematical definitions play in mathematics” (p. 412)—in the culture of mathematics definition is used to stipulate meaning, whereas dictionaries report on existing usage. More generally, Baker and Campbell (2004) found many students “either unable to comprehend the purpose of this type of writing [proof] or unclear on how to proceed in constructing such arguments” (quoted in Selden, Selden, & Benkhalti, 2017). It is this cultural dislocation—not the challenge of conceptual complexity—that animates proof pedagogy.

Although the distinction between understanding (or generating) a proof versus being adapted to proof as an activity system would seem to be straightforward, the history of psychology tends to obscure rather than enhance the boundary. This is particularly true of Piagetian theory, which initially encompassed both *microschemes*, which are “‘content’ oriented” (i.e., related to concepts) and *macroschemes*, which are “‘thought’ oriented” (i.e., oriented to practices) (Cobb & Steffe, 1983, p. 87). However, when Piaget’s macrogenetic stage theory encountered problems of décalage or unevenness of expression, researchers refocused developmental theory on microschemes (Steffe & Kieren, 1994).

These general developments notwithstanding, studies of proof sometimes apply Piagetian theory to practices of reasoning as illustrated in the Tall et al. (2012) study of “how individuals develop ideas of proof … and how these ideas change in sophistication over the long term” (p. 13) as the basis for development of proof competencies. Lest there be any doubt that the intended framework is Piagetian, this is made explicit later through reference to his stage theory: “In this section we discuss the general principles underpinning a learner’s path towards deductive reasoning from its sensorimotor beginnings, through the visual-spatial development of thought and on to the verbal formulation of proof” (p. 29).

Complementing the shifting boundaries of developmental psychology is a tendency for cognitive psychologists to restrict cultural learning to the affective domain. This is illustrated in the Greeno, Collins, and Resnick (1996) cognitive rubric which combines “general cognitive abilities, such as reasoning, planning, solving problems, and comprehending language” with “understanding of concepts and theories in different subject matter domains” (p. 16). Correspondingly, their situative domain (cultural learning) is restricted to a “focus on processes of interaction of individuals with other people and with physical and technological systems” (p. 17). As illustrated in Figure 1, the genres framework—to be discussed in the next section—redraws the boundaries in favor of a more extensive role for cultural learning.

<table>
<tr><td colspan="4">Greeno, Collins, and Resnick (1996) model</td></tr>
<tr><td>BEHAVIORIST APPROACH</td><td colspan="2">COGNITIVE APPROACH</td><td>SITUATIVE APPROACH</td></tr>
<tr><td>skills</td><td>subject matter concepts</td><td>general cognitive abilities</td><td>processes of interaction</td></tr>
<tr><td>HABITUATION</td><td>CONSTRUCTION</td><td colspan="2">ENCULTURATION</td></tr>
<tr><td colspan="4">Genres framework</td></tr>
</table>

Figure 1. Alignment of the genres framework with a standard learning theory framework

The Genres Approach

The genres approach parses the domain of learning into three constructs—habituation, construction, and enculturation—corresponding, respectively, with the three identified learning goals of teacher education: skills, knowledge (concepts), and dispositions (cultural practices) (Council for the Accreditation of Educator Preparation [CAEP], 2016). The principal innovation of the genres approach is that each of these learning constructs is conceived as independent of the others, neither in conflict for supremacy, nor as facets of a multifaceted whole. This enables a clearer and cleaner focus on pedagogical implications, organized as independent genres of teaching. As indicated in the foregoing discussion, we consider the teaching of proof (separate from the teaching of proofs) to be an enculturational learning goal, so we examine only that theorization of learning and its associated genre of teaching, here (see Kirshner, 2016, for a more extensive treatment).

Learning as Enculturation: I take enculturation to be the process of acquiring culturally particular ways of engaging with people, problems, artifacts, or oneself through enmeshment in a cultural community. Internalization of culture is vigorously theorized in the work of Soviet-era psychologist Lev Vygotsky; however, the sociogenetic orientation of his work is complicated by a more comprehensive ambition to account for "the dual process of shaping and being shaped through culture" (Cole, 1996, p. 103). Therefore, we supplement sociocultural theory with the sociological theory of Parsons (1951) who identified the unconscious and automatic character of enculturation as fundamental:

> Perhaps the most important distinction is between that attitude of expediency at one pole, where conformity or non-conformity is a function of the instrumental interests of the actor,

> and at the other pole the 'introjection' or internalization of the standard so that to act in conformity with it becomes a need disposition in the actor's own personality structure … The latter is to be regarded as the basic type of integration of motivation with a normative pattern-structure of values. (p. 37)

Teaching cultural practices. Parson's distinction sets up two subgenres for teaching of cultural practices. When the student is self-identified with the reference culture and seeks to *acculturate* her or himself to it, then the teacher need only model mature practices of the culture, and perhaps coach the student's performance. Because adopting those cultural practices fulfills the "instrumental interests" of the student, she or he will emulate the teacher's model. This subgenre is called *Acculturation Pedagogy* (Kirshner, 2016).

A more delicate and complex *Enculturation Pedagogy* is needed for the typical case in which students are not self-identified with the target culture. In this pedagogy, the instructional focus is on the classroom microculture, which the teacher seeks to shape so that it comes to more closely resemble the reference culture with respect to the target practices. The students learn from their immersion in the classroom community, not directly from the teacher whose influence must be unobtrusive. As Yackel and Cobb (1996) put it, desired practices eventually come to be "interactively constituted by each classroom community" (p. 475). The teacher exercises influence on the classroom microculture through subtleties of attention and encouragement that, over time, exert considerable influence on the modes of engagement manifest within the classroom microculture (Vygotsky, 1926/1997).

Enculturational teaching of proof. Enculturation pedagogy is relatively easy to enact in case the target cultural practices sometimes manifest themselves spontaneously. But significant enculturational goals that are remote from the students' current practices require a progressive agenda; modes of engagement initially encouraged by the teacher reach a level of general currency in the classroom microculture, eventually to be replaced by yet more sophisticated forms of engagement. Supposing, for instance, one wishes to teach the characteristic mode of argumentation known as mathematical proof (Pedamonte, 2007; Stylianides & Stylianides, 2009) to young students who typically support their arguments with reference to the authority of textbook and teacher—Harel and Sowder (2007) refer to these as "external conviction proof schemes" (p. 809). By betraying signs of interest whenever internal conviction arguments are offered (regardless how unsophisticated), the teacher may gradually shift the norms of classroom argumentation toward “empirical proof schemes,” even though what ultimately is sought are “deductive proof schemes” (p. 809). As in this case, learning of significant cultural practices may require a coordinated effort over months and years.

A Spyglass into the State of the Art of Proof Teaching

The genres framework of learning theories and pedagogical methods can be used as a tool to analyze and evaluate instances of curriculum (Kirshner, 2016). We conclude this paper by applying the genres lens to a pedagogical method of teaching proof described in Selden, Selden, and Benkhalti (2017).

Selden, Selden, and Benkhalti (2017) introduces a tool they developed in the context of a transition-to-proof course offered to sophomore mathematics students. This tool is a *proof framework*, “a way of structuring a proof, in which the student begins by writing the first and last lines of a proof and works towards the middle” (p. 3). Proof frameworks are needed because of “students’ tendencies to write proofs from the ‘top-down’ and their reluctance to unpack and use the conclusion to structure their proofs” (p. 2).

The top-down tendency would seem to be a case of too little exposure to authentic proof culture. When one reads a proof, one reads it sequentially. But the mental processes in creating one, do not match the written form. In fact, the proof framework is a format designed to simulate the way in which mathematicians actually construct proofs, which involves reasoning forward from conditions and backward from conclusions.

From this brief description, we see that the pedagogy is acculturationist, but with the teacher utilizing coaching rather than modeling. Indeed, the intent is to get “students started. Once they get started and begin to write straightforward proofs, they often get a feeling that writing proofs is something they really *can* do”—these are coaching motives. Enculturationist pedagogy would take a quite different path in which the teacher works with the current practices of the students, rather than direct them to mature practices. As these students are mathematics majors, it is reasonable to expect they are identified with mathematics culture, and hence suitable candidates for acculturation pedagogy.

What stands out in Selden, Selden, and Benkhalti (2017) is the energy the instructors have put into the coaching function in terms of analyzing the actual practices of proof: “We regard the proving process as a *sequence of mental* (e.g., ‘unpacking’ the meaning of the conclusion) or *physical* (e.g., drawing a diagram) *actions*. Such a sequence of actions is related to, and extends, what has been called a 'possible construction path' of a proof" (p. 3). This energy is reminiscent of the extensive research that Polya (1957) conducted into the heuristic methods by which mathematicians solve problem and prove theorems. However, Polya's engagement with students was based on the idea that "the teacher should help the student discreetly, *unobtrusively*. … ask a question or indicate a step that *could have occurred to the student himself*" (p. 1) while engaged in actual problem solving. This illustrates his commitment to enculturationist pedagogical methods that instill desired practices through cultural enmeshment rather than through didactic guidance. Unfortunately, this enculturationist thrust of his teaching was subverted by textbooks that explicated and practiced the various heuristics thereby "reduc[ing] the rule-of-thumb heuristics to procedural skills" (Stanic & Kilpatrick, 1988, p. 17).

In this paper we have sought to illustrate through the example of proof, how the genres approach brings clarity and power to pedagogical intentions through its alignment of theory with practice.

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UNDERSTANDING ASSESSMENT WHILE DEVELOPING EQUITABLE TEACHING PRACTICES

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Our research focuses on a growth model of teachers' ability to assess student learning as a result of creating equitable instruction for students in informal school settings. We describe data collected as part of a study examining the mathematical reasoning of Grades 3–5 students. Our research context took place in six elementary schools from rural and urban settings. Here, we focus on one of the schools by describing how a teacher began her instruction and over time, how she developed her assessment strategies to ensure that students obtained access to and support for algebraic reasoning, mathematical content, and discourse.

Equity research in mathematics education has attracted considerable attention in recent years (e.g, D'Ambrosio et al., 2013). Equity can be broken into multiple perspectives such as: cultural content, social organization, and cognitive resources (Brenner, 1998). While our foci may include all three, we study equitable practice in mathematics classrooms that centers on a growth model which highlights how teachers can progress in their disposition toward mathematical content and discourse. The research question in this study is how does equitable teaching affect teachers' assessment and instructional practices.

Related Literature

Research on Equity

Equitable instruction or practice in the mathematics classroom is defined as "those teaching practices that create fair distribution of opportunities to learn mathematics among students, with special emphasis on the learning of students who are members of ethnic and social groups currently 'underperforming' in mathematics, and those students who depend on schools for their primary access to learning" (Goffney, 2010, p. 7). Banks (2001) also states that equity is utilizing various teaching strategies and creating a classroom environment that helps students from diverse racial, ethnic, and cultural groups attain the knowledge, skills, and dispositions needed to function effectively within society. Goffney (2010) and Banks (2001), among many researchers, argue against deficit models in equity research, aligning with our beliefs and experiences in mathematics classrooms.

Research on Assessment as Related to Equity

Assessment is a way to evaluate whether students and teachers meet a target goal or learning outcome. Not only does an assessment determine the outcome of an event, it also informs a teacher of two items: 1) what a student can do on a particular problem, concept, or task, and 2) how does what the student knows affect instruction. The assessments in this study are formative assessments that inform teaching and learning versus a summative assessment to assign a score to determine one's performance.

The On Track-Learn Math research project provided a unique space for teachers to examine their assessment and instructional practices because the teachers taught in a nontraditional setting, an after school program. Teachers could experiment with non-routine problems and utilize different assessments to determine student learning which allowed them to begin to adjust their instructional practices. The research team utilized the Structure of Observed Learning Outcome (SOLO) taxonomy (Biggs & Collis, 1982) to examine what teachers and students could do (process, conceptual, and discourse) on a task, which led the teachers to develop equitable practices over time.

Theoretical Framework

Many researchers have found that the quality of instruction is directly related to teacher knowledge and student achievement (Darling-Hammond, 1999; Ingersoll, 2002; Whitehurst, 2002). However, culture plays an important role in the academic development of students. Culture can be conceptualized as the "combination of norms, values, beliefs, expectations, and conventional actions of a group" (Phalen, Davidson, & Cao, 1991). Culture is a dynamic construct which influences how and what knowledge is produced while also defining important differences among learners (Grimberg & Gummer, 2012). Also, students make gains when they have a quality teacher (Hill, Rowan, & Ball, 2005) – one who can successfully choose a task, provide rich instruction, and orchestrate meaningful discourse. However, there exists a paradox of accessible, equitable and successful teaching, learning, and assessment outcomes for all students. Here, we posit a possible way to address such a paradox by using a sociotransformative constructivist perspective.

This study uses sociotransformative constructivism as the theoretical lens to guide inservice teachers' use of assessing diverse and multicultural students in an after-school program. Sociotransformative constructivism (Rodriguez, 1998, 2010, 2015) merges multicultural education and social constructivist theoretical frameworks as a theory for learning and teaching. Rodriguez (1998) describes the four components of sociotransformative constructivism: (a) dialogic conversation (b) authentic activity (c) metacognition (Idol & West, 1991) and (d) reflexivity. These components are meaningful interactions that evolve organically and are facilitated by teachers. Sociotransformative constructivism assists teachers in becoming more aware of how issues of power, gender, and equity influence who has access to education, and the influence each has over what and how subject matter is taught and assessed.

The Structure of the Observed Learning Outcome (Biggs & Collis, 1982) is a taxonomy for assessing students' understanding of a given task. The SOLO taxonomy was designed to empower teachers to apply theoretically based knowledge of student thinking and learning so their teaching practices would maximize student achievement. SOLO also merges well with the sociotransformative theoretical framework because it provides transitional movement for students and teachers to deepen their level of thinking through a cultural and equitable lens. The SOLO taxonomy approaches assessment as an ongoing process by informing instruction using the prestructural, unistructural, multistructural, relational, and extended abstract stages (see Table 1). The stages in the taxonomy take students from knowing one point about a problem to knowing multiple points about a problem and describing their thinking and finding patterns, to finding multiple solutions or strategies and rules.

Table 1

The SOLO Taxonomy

Pre-structural	The task is not attacked appropriately; the student hasn't really understood the point and uses too simple a way of going about it.
Uni-structural	The student's response only focuses on one relevant aspect.
Multi-structural	The student's response focuses on several relevant aspects but they are treated independently and additively. Assessment of this level is primarily quantitative.
Relational	The different aspects have become integrated into a coherent whole. This level is what is normally meant by an adequate understanding of some topic.
Extended abstract	The previous integrated whole may be conceptualized at a higher level of abstraction and generalized to a new topic or area.

In the On Track project, a type of question asked was, "If 100 square tables can seat 202 people, how many people will be able to sit at 101 tables?" The purpose of this type of question was to deepen the student's understanding regarding the total number of square tiles at a particular phase. The highest level of the SOLO (deep understanding and extending) in this example would ask the question, can you develop a rule? As tasks become richer in nature, the idea is that discussion from students deepens and the understanding between teachers and students also deepens. This also aligns with the elements of the sociotransformative constructivism framework.

Here, our research focuses on how one teacher increased her understanding of assessment as she developed equitable teaching practices over time. We describe how one teacher began her mathematical instruction and how she varied/increased her assessment strategies through growth in her practices to ensure that students obtained access to the algebraic reasoning, mathematical content, and discourse. It is also our intent to bring to the forefront how classroom instruction that balances the structures found in the sociotransformative framework and elements of the SOLO taxonomy to assess student learning and produce equitable teaching and assessment practices.

Methodology

Design and Subjects

The On Track project included students in grades 3, 4, and 5 and took place in six elementary schools (some Title 1) located in both rural and urban settings in the eastern part of the United States over the course of two years (4 semesters). Children attended 10 sessions per semester, lasting two hours per day, twice a week. Professional development sessions were held at the beginning of each semester for one lead teacher and one assistant teacher per school. Ongoing and real-time professional development was offered by the research team as needed during the filming of sessions. Each session took place directly after school in classrooms of the lead teacher. However, in this study we investigate one teacher in one of the schools from the larger data set and refer to each semester as a cycle.

Task and Instruction

Students, mostly in grade 3, sat in groups of four for this study, and the tasks given included a series of algebraic reasoning questions centering on functions. For example, "This machine has a rule that makes new numbers. Your job is to guess the rule." The question goes on to show an input of 1, output of 1; input of 2, output of 4; input of 3, output of 9. Each student had a copy of the problem to write on; however, students were encouraged to discuss, share ideas (even work together). As they worked the problems, the lead teacher and supporting teacher circulated around the classroom offering assistance as needed. For more information about the tasks themselves, Store (2013) details the nature of such tasks.

Evidence and Analysis

The SOLO taxonomy was used to examine one teacher's (Ms. Pearson, a pseudonym, also referred to as the Lead Teacher) equitable assessment practices over the course of four cycles of problem solving in an after-school program of students mostly in Grade 3 for this analysis. Unlike other taxonomies, the SOLO taxonomy was chosen because it was designed to measure the level of what students are able to do or "learn (to do)" (Brabrand & Dahl, 2009). Data collected included video, student work samples, observations, and interviews. Video data was viewed separately by each researcher, analyzed, and viewed again together - all using the SOLO taxonomy during the process. Attention was given to the types of questions the lead teacher asked the students during her work with the tasks. We coded the questions asked by the teacher using the phases of the framework (prestructural, unistructural, multistructural, relational, and the extended abstract).

We also performed content validity through selecting algebraic tasks that have been measured valid and reliable from experts. The results from the tasks were triangulated through the authentic work samples, observations, and interviews. Furthermore, the researchers implemented the problems with the teacher participants and determined their level of understanding through the SOLO taxonomy, and had discussion with the teacher to validate the results. Then, the teacher utilized the same content with Grade 3, 4, and 5 students in the after-school program, and these data were collected and analyzed.

When we first started the lessons, the teacher was given a script to follow to allow her time to become comfortable with the algebraic content. Our intent was for all the teachers in the project to veer off the script once they became use to the types of tasks and the style of student engagement in the tasks. During the first cycle, Ms. Pearson followed the script verbatim. She asked a question, waited for an answer, and then moved on to the next question. For example, Ms. Pearson first started the initial set of tasks with unistructural questioning like, “How many sides does it take to make one table?” She did grow to multistructural questions like, “How many sides are needed for each of the tables pictured?” but her assessment of the students’ answers was underdeveloped. If a child gave an incorrect answer, she moved on to someone who had the correct answer without inquiring about the processes of either child’s thinking.

At the second cycle we began to notice a significant change. Ms. Pearson engaged students in both large group and small group discussions. We found this to be an equitable teaching and assessment practice in conjunction with the more advanced phases of the SOLO framework. For example, she posed a relational question to the entire group, then she and her assistant teacher circulated around the room, spending 10-15 minutes with small groups allowing them to process and describe their strategies. She prompted the assistant teacher to attend to their pictorial representations and verbal descriptions. As if compiling data, she would then bring the class together as a whole, and allow them to present their solutions using the document camera. If a student had everything worked accurately, she would hold off on allowing them to share first to allow mistakes to be a part of every task given. In the act of sharing, many students stood at the front of the classroom and self-corrected their mistakes simply because they were allowed the space to do so.

As the project continued into the third and fourth cycles, Ms. Pearson often used relational questioning at thoughtful times *during* their presentations, as well. For example, when one student described their work with the pentagonal tables, she followed up by asking, “If 2 tables include 9 sides, how many sides will 10 tables include?” This challenging yet engaging style of questioning got the students excited because they were already invested in the problem. She assessed on the spot that the child was ready for a more sophisticated line of thinking about that problem.

Results and Discussion

All four phases of the SOLO taxonomy are reported during most of the teaching cycles in the On Track project along with equitable teaching and assessment practices. These, as we predicted, are difficult to separate due to the nature of a learning environment that creates opportunities for all students to make sense of the mathematical content. The very structure of the On Track project began with scripted lessons and a narrow focus, which we were concerned the teachers may not want to drift away from. One reason we scripted so much at first was because we observed Ms. Pearson and other teachers in the project using a lot of direct instruction in their regular classrooms. We wanted to start them out with a familiar format for the project. However, as Ms. Pearson met with us (after the second session was completed) for professional development, she started bringing in ideas about how the students were approaching the task. We showed video examples of reform-based classrooms and this inspired her to make changes for the third session as described in the above analysis.

Through our analysis of the On Track data over, we found that Ms. Pearson grew in her confidence, her ability and her content knowledge. Providing targeted professional development and allowing teachers to practice the learned strategies in a non-threatening environment supports the growth and success of teachers. Ms. Pearson also commented on how she began taking the formative assessment strategies back into her regular classroom to really analyze student thinking and understanding of the concepts. She believed these experiences were beneficial to the mathematical growth of her students that were not necessarily in the after school program. The research allowed the teachers to connect rich mathematical tasks to targeted learning outcomes, while teachers were able to strengthen their assessment strategies and utilize the knowledge they learned about the students' thinking for the following learning episodes, in this case the next after school session. It is difficult to capture how teachers assess student learning and utilize this in their teaching; however, Ms. Pearson demonstrates this ability, as she increased her questioning skills, the level of discourse in the classroom, and her ease and understanding of each of the tasks. Ms. Pearson began to think on a more global scale of how to transform the learning of her students with respect to the levels of formative assessment found in the SOLO taxonomy.

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IMPROVING TEST DEVELOPMENT REPORTING PRACTICES

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The purpose of this proceeding is to stimulate conversations about the Standards for Educational and Psychological Testing (AERA, APA, & NCME, 2014) within mathematics education research. First, I describe the five sources of validity evidence, validity arguments, and summary statements. Next, this proceeding provides guidelines for researchers intending to publish their work focusing on validation, specifically focusing on summary statements. Finally, an example summary statement is offered for readers.

Recently, *Investigations in Mathematics Learning* has published a special issue on validity and validation issues in mathematics education contexts but also currently welcomes manuscripts focused on validity, validation, and assessment in mathematics education. Measure quality strongly influences the quality of data collected and concomitantly, the findings of a research study (Gall, Gall, & Borg, 2007). This proceeding aims to stimulate conversations around research focusing on validation and sources of validity evidence for educational testing as it relates to mathematics education research. It also describes guidelines for reporting validation research with quantitative instruments. Moreover, this proceeding may serve the broader community as a means to communicate information to potential end users.

Related Literature

Standards for Assessment Development

The American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement Education (NCME) provide clear standards regarding measurement validity and reliability as it relates to education testing in *Standards for Educational and Psychological Testing* (2014). At a minimum, sufficient evidence from a subset of the five sources of validity must be shared: (1) evidence based on test content, (2) evidence based on relations to other variables, (3) evidence based on internal structure, (4) evidence based on response processes, and (5) evidence for validity and consequences of testing (AERA, APA, & NCME, 2014). The amount and types of evidence needed for a measure are predicated upon the nature and scope of the instrument (AERA, APA, & NCME, 2014). Unfortunately, "evidence of instrument validity and reliability is woefully lacking" (Ziebarth, Fonger, & Kratky, 2014, p. 115) in the literature. To make matters worse, evidence related to the validity of results of quantitative instruments are not necessarily conceptualized or defined consistently in the research literature (Bostic, 2017; Lissitz & Samuelsen, 2007; Mislevy, 2007). Put simply, there are standards for developing and engaging with educational testing that should be followed.

Beyond standards, there is also a stylistic means for communicating why the evidence is appropriate and how it addresses the standards. Kane (2001, 2012, 2016) suggests an interpretation/use argument and a validation argument within the context of a Toulmin (1958) style argument that includes evidence (datum), justification (warrants), and ultimately a claim. For instance, evidence for a claim might be that the associated Cronbach's alpha reliability of an instrument used with a representative sample in a particular context is 0.89. The justification connecting the evidence and claim might express that Cronbach's alpha values approaching 0.9 are considered excellent (Streiner, 2003). Thus, the claim might be that a particular instrument produces sufficiently reliable scores that can be used in research and evaluation in a particular context. Kane's style is not the only approach; Schilling and Hill (2007), Wilson (2003; 2017), and others are also viable. In the end, it is an instrument developer's responsibility to communicate the argument to potential users in a clear and effective manner so that they are fully cognizant of an instrument's scope (AERA, APA, & NCME, 2014; Wilson, 2017). Unfortunately, few articles describing an instrument's validity evidence go beyond reporting reliability or content evidence. If mathematics educators intend to conduct research that builds upon works of others and generalizes, then it is prudent to follow the *Standards* (AERA, APA, & NCME, 2014) and be transparent about data collection instruments used in research.

Justifying a Need for Validation Studies: An Example from Observation Protocols

Bostic, Lesseig, Sherman, and Boston (2017) conducted a wide search of mathematics education research published between 2000–2015 that drew upon classroom observations. One goal of this study was to examine the rigor of instruments used within classroom observation research. Manuscripts were drawn from first or second-tier mathematics education journals only (see Brigham Young University Department of Mathematics Education, 2008; Toerner & Arzarello 2012). While the 2014 *Standards* may not have impacted authors of papers from our sample; assuredly, the 1999 *Standards for Educational and Psychological Testing* (AERA, APA, & NCME) should have been followed. This search resulted in 114 manuscripts involving classroom observation but only 44 manuscripts used a classroom observation protocol (COP). Of the 44 manuscripts, 28 unique COPs were mentioned. Thus, the team sought out published, peer-reviewed manuscripts reporting validity evidence for these 28 COPs. In sum, 11 manuscripts reported validity evidence for eight COPs. Thus, it was concluded that 8 of 28 COPs protocols had any reported validity evidence found in peer-reviewed publications. This result is gravely concerning for many reasons. First, scholars need to know how and where instruments' results and interpretations are valid. If a researcher used an instrument in a way outside of the scope, then the results are susceptible to be invalid. Second, failure to report validity evidence prevents the greater community from taking up opportunities for further study. As Cobb (2007) expresses, the bricolage approach to research works effectively only when the scholarly community can confidently and justifiably nestle new knowledge alongside prior work. Third, instrument users need information to make qualified decisions about whether a particular instrument is appropriate for a given sample. For instance, both the Revised Standards for Mathematical Practice (SMPs) Look-for Protocol (Bostic & Matney, 2016; Bostic, Matney, & Sondergeld, 2017) and Mathematics Classroom Observation Protocol for Practices (Gleason, Livers, & Zelkowski, 2017) are appropriate for K-12 contexts. Both protocols focus on teachers' promotion of the Standards for Mathematical Practice; however, it is in the validation papers that one learns the former focuses on teachers' actions whereas the latter examines instructional actions that connect teachers and students. Thus, a small change in wording confers a major difference in the construct under observation. All of this leads to considerations of other commonly used quantitative instruments like content knowledge assessments for teachers and students as well as attitude and beliefs questionnaires.

Reporting Validity of Score Interpretations

There are not one-size fits all approaches to discussing validity of score interpretations from quantitative measures. However, this proceeding provides a few guidelines. In April 2017, 41 researchers attended the Validity and Measurement in Mathematics Education (V-M^{2}ED; National Science Foundation project #1644314) conference. A primary outcome from the conference was guidelines for writing summary (aka purpose) statements (Carney, Bostic, Krupa, & Shih, 2017). A secondary outcome was helping researchers better communicate validity information and arguments. Because validity arguments can take on a particular style, conference leaders and attendees agreed that an argument related to validity aspects should include three aspects. First, they should frame their chosen validity argument within a particular style (e.g., Kane, 2012, 2001; Schilling & Hill, 2007; Wilson, 2003, 2017). Second, they should briefly describe the chosen framework for the reader. This is important because readers are unlikely to be intimately familiar with a particular framework. Third, authors should provide sufficient evidence for their claims and purposes. This aligns with the *Standards* (AERA, APA, NCME, 2014), which indicates that instrument developers have sufficient evidence for a claim when they feel they have enough for a needed intent (e.g., research and evaluation purposes versus formative assessment). Inclusive of this third aspect is communicating a clear summary statement. Summary statements are written for the end user and briefly describe the scope and nature of an instrument. Fourth, authors should justify their evidence as being appropriate or fitting within expected guidelines. That is, readers ought to be able to connect evidence and claims through the justification. Finally, readers and instrument developers must re-evaluate evidence in light of new knowledge about a construct, using a tool in a different manner, setting, or with a different sample, and/or if revisions or alternate forms are necessary.

A challenge for authors is reporting validity evidence through a coherent validity argument in page limits set by journals. How might authors respond to daunting page limits? For instance, *Investigations in Mathematics Learning* has a 20-page limit, which makes it difficult to convey all of the information in a meaningful way for potential users. Broadly speaking, there are two approaches: (a) Presenting a comprehensive validity argument related to five sources of validity, or (b) Communicating a more detailed and focused validity argument for a subset of the five sources of validity. Bostic, Matney, and Sondergeld (in press) use a comprehensive approach by sharing evidence for all five sources. In this manuscript focused on the Revised SMPs Look-for Protocol, the authors make a roadmap of claims, draw upon various forms of evidence from quantitative and qualitative research, and connect the evidence and claims using expectations for acceptability of evidence (i.e., justification). On the other hand, Kosko (2017) published a manuscript that discusses a quantitative measure focused on three sources of evidence: test content, response processes, and internal structure. In both instances, the reader learns about the assessment, its purpose and focus, and how claims, evidence, and justification are linked for facets of an assessment. No matter what style is selected, a summary statement must be communicated. A first start for any manuscript describing an instrument and presenting a validity argument, is writing a summary statement.

Getting Started: Summary Statements and an Example

A summary statement describes the purpose of a quantitative instrument in a brief manner in such a way that an end-user is able to determine whether a particular instrument is appropriate for a desired need. At V-M^{2}ED, a series of questions were discussed (see Figure 1). As a result of exploring these questions, attendees and leaders determined that effectively answering these questions was necessary for a summary statement.

- What is being measured?
- Why is it important to measure?
- How is it being measured?
- Who is being measured?
- What is the context for the measurement?
- How might scores be interpreted?
- How might scores be used?

Figure 1. Questions for purpose statement.

It may be unclear how to respond to these questions hence, an example summary statement is shared below for the Problem-solving Measures (i.e., PSM6, PSM7, and PSM8; see Bostic & Sondergeld, 2015; Bostic, Sondergeld, Folger, & Kruse, 2017).

The Problem-solving Measures (PSMs) measure an individual's problem-solving performance related to the Common Core State Standards for Mathematics (CCSSM). The CCSSM include both content and practice standards, which the PSMs address. Specifically, the PSMs are connected to the sixth-, seventh-, and eighth- grade content standards. Problem-solving performance in relation to content standards is important because problem solving is mentioned in every domain of the CCSSM as well as Standard for Mathematical Practice #1 (SMP1; i.e., Make sense of problems and persevere in solving them). Therefore, problem solving is a central feature of the CCSSM. The PSMs are a series of constructed-response word problems. Each PSM contains 15 items representing the five CCSSM domains in these grade levels; there are three items connected to each domain. The PSMs are intended primarily for students in grades six, seven, and eight. Additionally, the PSMs may also be administered to preservice mathematics teachers. Generalizability evidence for both contexts is available. It takes students approximately 75 minutes to complete the measure whereas preservice teachers usually need 45 minutes. There are paper-and-pencil and online versions of the PSMs. No matter the format, paper, pencil, and calculators may be used while testing. Responses are scored dichotomously. In past research, scorers are trained; however, there is potential for individuals (i.e., classroom teachers, school psychologists, or other researchers) to receive training that aligns their scoring with past trained scorers. Results are interpreted in two ways. The first is using Rasch measurement. Through Rasch, a set of results can be input and a class report may be shared—indicating how students' performance compares to the mean. A second way is using raw scores and interpreting performance based on past performance. Both are valid and appropriate score interpretation methods but lead to different interpretations. Raw scores indicate rates of correct responses whereas results for a class (or individual) analyzed with Rasch modeling communicate the individual or class' performance in comparison to average (mean) ability. Obviously, there are differences in expected outcome scores for students and preservice teachers, means are higher for preservice teachers. Scores should be used formatively to provide data for students, teachers, parents/guardians, and other educational stakeholders. Ultimately, the scores may demonstrate longitudinal (i.e., multi-year) growth. Scores may also be used as evidence of students' outcomes within the contexts of teacher-focused professional development or other research foci. Finally, scores from preservice teachers intending to teach sixth, seventh, or eighth-grade may be interpreted in light of their pedagogical content knowledge. To be clear, scores from students should not be used to move them into different mathematics courses or other high-stakes fashion. Similarly, scores from preservice teachers are not indicative of their mathematics content knowledge but instead, pedagogical content knowledge. The PSMs address content that preservice teachers will teach students, not strictly general content knowledge. Those interested in using the PSMs should contact the measure authors because cost is dependent upon needs related to the PSMs. It should be evident from this summary statement what the PSMs measure and how they measure outcomes.

Conclusion

The purpose of this manuscript was (a) to provide readers with a sense of the five sources of validity evidence, (b) to provide guidance for writing manuscripts focusing on assessment and validation, and (c) to offer an example summary statement that might serve as a model for future authors. In the last two years, there are more validity studies being published in *Investigations in Mathematics Learning*. No matter authors' choices to frame their validation studies, a central focus must be the *Standards* (AERA, APA, & NCME, 2014) and communicating information in a clear, coherent manner that validates the outcomes and interpretations from the instrument under discussion.

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DRAW YOURSELF DOING MATHEMATICS: VALIDATING AN INSTRUMENT AND A RUBRIC

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The Draw Yourself Doing Math drawing prompt and accompanying rubric is being studied to establish reliability and validity of this tool to understand how students view mathematics. This report details the efforts made to clarify the drawing prompt phrasing and protocol, the process of identifying and describing rubric categories, and the future plans for establishing coding reliability and instrument validity.

Methodology

The Draw Yourself Doing Mathematics prompt (Bachman, Berezay, &Tripp, 2016) was developed to assess undergraduate student views toward mathematics. This piece studies the reliability and validity of the prompt and scoring rubric. The study is guided by the seven steps to validating education instruments given by Artino, La Rochelle, Dezee, and Gehlbach (2010). These seven steps are (a) conduct a literature review, (b) conduct interviews and/or focus groups, (c) synthesize the literature review and interviews/focus groups, (d) develop items, (e) conduct expert validation, (f) conduct cognitive interviews, and (g) conduct pilot testing.

Conducting a Literature Review

In this stage, the researcher "ensures that the construct definition aligns with relevant prior research and theory and identifies existing survey scales or items that might be used or adapted" (Artino et al., 2010, p. 464). The drawing prompt was first developed to assess changes in student views toward mathematics in an experimental general education course pairing mathematics and dance (Bachman, Berezay, & Tripp, 2016). Because of the multimodal approach of the course, the researchers wanted to find a way to assess students' pre and post views about mathematics that would describe differences in student views about where, with whom, and how the doing of mathematics happens. At this time, researchers learned about the use of drawing prompts to understand preservice teacher views toward learning and teaching mathematics (Mcdermott & Tchoshanov, 2014). The researchers then conducted a literature review to examine how drawing prompts were being used to understand student views toward mathematics. This review showed that all drawing prompts being used in mathematics were to understand preservice teacher views of mathematics, mathematics learning, and mathematics teaching (Burton, 2012; Mcdermott & Tchoshanov, 2014; Utley, Reeder, & Redmond-Sanogo, 2015). Therefore, a new prompt was needed to understand student views of mathematics. To date, this study has focused on undergraduate student views of mathematics, though local teachers have also experimented with giving the prompt to K-12 students.

To prepare for a conversation with a focus group of RCML participants, the researchers collected preliminary drawings based on the prompt "Draw yourself doing mathematics. *Don't worry about the quality of the drawing. Just sketch what comes to mind."* (Bachman, Berezay, & Tripp, 2016). The research team developed an overall impression score that assigned a numerical value from 1 (severely negative) to 7 (extremely positive) and an accompanying rubric (Bachman, Berezay, & Tripp, 2016). In this study, the treatment group had a mean increase of 2.25 points (and median increase of 2 points) on this overall impression score; the control group showed a mean decrease of 0.22 points (and median change of zero points) (Bachman, Berezay, & Tripp, 2016). The pretest drawings in both classes and the posttest drawings in the control class "depicted widespread views of mathematics as an unpleasant endeavor, navigated alone, studied at desks, and plagued by unproductive struggle" (p. 56). Based on the rubric categories of locating, appearance, and activity used by Farland-Smith (2012) and insight from prevalent themes in these drawing, the researchers identified four potential future rubric categories: location, activity, others, and affective state.

Conduct Interviews and/or Focus Groups

In this stage, the researcher requests input on the formation of the newly developed instrument from the population of potential users (Artino et al., 2010). The results on this preliminary study were presented at the 2016 RCML Annual Conference to generate feedback on the prompt itself and the analysis of the resulting drawings. The experts in attendance generated three main questions. First, the attendees wondered about the phrasing of the prompt. Since almost all of drawings featured only one person in the drawing (the student), the attendees wondered if this could be attributed to the fact that the prompt asks the student to draw *yourself* doing mathematics and suggested testing a prompt that did not include mention of self. Second, the attendees wondered whether the limited time given to complete the drawings contributed to nature of the drawings. It was suggested to give the prompt as a take home assignment so students had more time to think about what they wanted to draw. Third, the attendees wondered if the range of responses would be broader than the overall impression could measure if students were polled from different mathematics classrooms and suggested giving the prompt to students in a range of mathematics classrooms. The attendees were supportive of the proposed future rubric categories.

Synthesize the Literature Review and Interviews/Focus Group

Here the researcher decides how to incorporate ideas from the focus group (Artino et al., 2010). To research the suggestions given at RCML, the researchers administered the prompt in 12 mathematics classrooms with total enrollment of 357 students. The courses represented were prealgebra, introductory probability and statistics, trigonometry, precalculus, calculus, mathematics for elementary teachers, linear algebra and differential equations, foundations of algebra, and number theory. A new prompt was also tested that read, "Draw what comes to mind when you think of doing mathematics." Each course section was randomly assigned to one of four groups: (a) original prompt given in class, (b) original prompt given as a take home task, (c) new prompt given in class, or (d) new prompt given as a take home task.

Prompt wording. Table 1 shows the comparison of drawings from each of the two prompt types. The proportion of the drawings that featured more than one person was 0.08 for the original prompt and 0.10 for the new prompt. A chi-square test for differences in proportion showed no significance difference in the proportion of students that drew someone other than themselves in the drawings, $\chi^2(1, N = 195) = 0.234, p = 0.6284$. However, the researchers did notice that the proportion of drawings that featured no people seemed higher when using the new prompt. Table 1 reports that the proportion of drawings with no people was significantly higher when using the new prompt, $\chi^2(1, N = 195) = 56.152, p < 0.0001$. The researchers concluded that the original prompt was preferred because it did not affect the degree to which students drew other people other than themselves, but the new prompt did affect whether or not any people were drawn in the picture.

Table 1

Comparison of Prompts

		With others		With no people	
	n	Number	Proportion	Number	Proportion
Original Prompt	91	7	0.08	3	0.03
New Prompt	104	10	0.10	54	0.52*

*p< 0.0001

Method of administering. When administering the prompt in class and as a take home task, the researchers looked at (a) completion percentages, (b) drawing details, and (c) overall impression scores. To measure the completion percentages, the researchers kept track of how many students were enrolled in each section the prompt was given in and how many prompts were completed. Table 2 summarizes the proportion of the class that completed the prompt when given as an in-class task and when given as a take home task. While the course instructors were asked not to given any extra credit for completing the prompt, one instructor did award extra credit to both sections of his course that completed the prompt as a take home assignment. A chi-square test for differences in proportion showed no significance difference in the proportion of students completing the prompt in class and as a take home task, $\chi^2(1, N = 357) = 1.892, p = 0.1689$. However, the chi-square test for differences in proportion did show a significant difference in the proportion of students completing the prompt as a take home task with no extra credit and those completing it in class, $\chi^2(1, N = 282) = 28.949, p < 0.0001$. There was also a significant difference between the completion proportion of students completing the task as a take home assignment with and without extra credit $\chi^2(1, N = 207) = 11.940, p = 0.0005$.

Table 2

Completion Percentages and Overall Impression Scores for Completion Type

				Overall Impression	
	n	Proportion Completed	n Original Prompt Completed	Mean	Std. Dev.
In class	150	0.7	38	3.44	1.67
Take home	207	0.55	64	3.39	1.36
Take home – no extra credit	132	0.38	17	3.89	1.17
Take home – extra credit	75	0.63	47	3.18	1.4

To compare the details in the drawings, the researchers chose to analyze only the drawings that used the original prompt as this prompt was shown to be preferred by this point in the analysis. Comparison of the drawings from the three groups showed most detail drawn in the responses from students completing the task at home with extra credit, fewer details were drawn when completed in class, and the fewest details were drawn when completed at home with no extra credit. While differences in details did arise, there were no significant differences in the resulting score using the overall impression scale between the completion types (in class and take home $t(100)=-0.174$, $p=0.8619$, in class and take home with no credit $t(53)=1.004$, $p=0.3200$, in class and take home with extra credit $t(83)=-0.781$, $p=0.4371$), take home with and without extra credit $t(62)=-1.866$, $p=0.0668$). The researchers therefore concluded that giving students 10 minutes in class to complete the prompt is an acceptable way to administer the prompt.

Range of courses. There were no drawings collected from the 12 classes that the overall impression scale would not describe despite the fact that drawings had been collected from a much wider range of mathematics courses. The researchers used this wide range of responses to help shape the rubric descriptions for the new scoring categories of (a) location, (b) others, (c) activity, and (d) affect.

Develop Items

The task in this stage is to design items that are clear and understandable (Artino et al., 2010). Only one adaptation was made to the original overall impression scale in this stage, and that was to create a "cannot be categorized" rating of "0" for drawings that cannot be assessed. This decision was made because of a drawing collected from a kindergarten classroom by Mkina (2017). It is recommended that this category be removed before calculating any statistical analysis on the scores of drawings when using the prompt in a study.

The researchers created four rubric categories for scoring drawings: location, activity, others, and affective state. The development of these four scoring categories rest on the prior work of Farland-Smith (2012) who scored the categories of appearance, location, and activity using the scale of "0" for "cannot be categorized," "1" for "sensationalized," "2" for "traditional," and "3" for broader than traditional. Because the researchers wanted to be able to count the number of drawings that featured broader than traditional classrooms from the use of mathematics outside the classroom, the scale was extended to five scoring possibilities in all the categories except for affect: 0 – Cannot be Categorized, 1 – Sensationalized, 2 – Traditional Learning, 3 – Broader than Traditional Learning, 4 – Real World. The affective state category has the distinctions 0 – Cannot be Categorized, 1 – Negative, 2 – Neutral, and 3 – Positive.

Table 3 shows the inter-rater agreement for each of the four rubric categories and the overall impression scale before and after discussion between two raters. Following the discussion, the rubric descriptions were edited to better support future inter-rate agreement.

Table 3

Inter-rater Agreement Before and After Discussion

	Before	After
Location	0.96	1.00
Activity	0.78	1.00
Others	0.94	1.00
Affective	0.86	0.98
Overall	0.56	0.96

Conduct Expert Validation

The goal of this stage is to assess how clear and relevant the items are with respect to the construct of interest (Artino et al., 2010). At the 2017 RCML Annual Conference, the researchers reported on the work done to date to develop and test the drawing prompt and scoring rubric. The feedback from this session centered on next steps needed for validity and reliability testing. The experts suggested conducting interviews with participating students as a way of checking the rubric coding and gathering attitudinal survey data on the participants to study the correlation between validated survey results and the scores from this prompt.

Conduct Cognitive Interviews

At this stage, the researcher seeks input on how respondents interpret items to make sure items are being interpreted in the manner the instrument designer intended (Artino et al., 2010). In the final stage of data collection still to come, participating students will be asked to volunteer for a one-hour interview about their drawing and attitudinal survey. A subset of the volunteers will be selected so that each overall impression score is represented among the interviewees. During these interviews, researchers will ask participants to describe their drawing to check for consistency in how the researchers coded the drawings. The interviewer will describe the drawing using phrases from the rubric to achieve member checking with the students about how the drawings are being interpreted. For example, a drawing coded as Location – 2, Activity – 1, People – 1, Affective – 1, and Overall Impression – 2, the interview would ask the student to comment on the degree to which they agree with statements like “Doing math happens at a desk, involves a lot of unproductive struggle, occurs alone, and is a negative experience for me.”

Conduct Pilot Testing

In this final stage of validation efforts, “the researcher checks for adequate item variance, reliability, and convergent/discriminant validity with respect to other measures” (Artino et al., 2010, p. 464). In the final stage of data collection yet to come, students from a range of classes on campus will complete the prompt and the Attitudes Toward Mathematics Inventory (ATMI) developed by Tapia and Marsh (2004) to research the content validity of the prompt and rubric. The ATMI was chosen to use in this study because it is a validated instrument (Majeed, Darmawan, & Lynch, 2013; Tapia & Marsh, 2004) averaging around 20 minutes for implementation time (Tapia & Marsh, 2004).

Following the collection of these drawings and surveys, researchers will code the drawings using the rubric categories of location, activity, others, affective state, and overall impression. In this coding process, researchers will be measuring the reliability of the rubric using Cronbach's alpha coefficient and conducting a factor analysis of the four subcategories and the overall impression scale. Correlation analysis will also be conducted between drawing prompt scores and the attitudinal survey scores.

Further study and Implications

Results of the cognitive interviews and pilot testing for reliability and validity will be shared at the 2018 RCML Annual Conference. Future work will still be needed to validate the prompt and rubric for use in K-12 classrooms as this validation effort has been focused on university undergraduates. Additionally, the prompt and rubric will still need to be tested for reliability and validity at other postsecondary institutions across the United States and in international settings. The implication of this finished work is the creation of a reliable, validated survey alternative for generating information on student views toward mathematics that can be administered in a few minutes. The instrument has great potential for use with students lacking the literacy skills needed to take a survey (i.e., age, English Language Learners, and students with learning differences).

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PROSPECTIVE TEACHERS DEVELOPMENT OF NUMERICAL REASONING

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Ninety-eight prospective elementary teachers (PT) participated in a mixed-method intervention study aimed at enhancing their development of numerical reasoning. This study examined the extent to which PTs learned to notice, state, and apply the commutative property, the associative property, and the distributive property. Treatment groups received a series of workshops focused on the development of algebraic reasoning. Pre-and post-teaching assessment of learning occurred over a period of four weeks, with data gathered from pencil-and-paper tests and one-to-one interviews. Most PTs showed large gains in their understandings of algebraic structure and number properties; effect sizes of the intervention were large.

Background of the Study

The mathematics component of the common core curriculum (CCSSM, 2010) emphasizes the importance of developing algebraic reasoning in the elementary and middle grades. The NCTM (2000) Standards suggest that students should be able to “identify such properties as commutativity, associativity, and distributivity, and use them to compute with whole numbers” in Grades 3–5 (p. 158). However, typical U.S. computational lessons often emphasize steps that lead to quick answers and deemphasize a deep understanding of the underlying properties (Schifter, 1999; Thompson, 2008).

Carpenter, Franke, and Levi (2003) stated that children have a great deal of implicit understanding of the properties of basic operations, which can be seen when one examines their invented algorithms to solve problems involving basic operations. For example, to compute $(3 \times 4) \times 25$, a student may use the associative property (AP) to compute the later two numbers $3 \times (4 \times 25)$ and find the answer easily (NRC, 2001). These structural elements of numbers might serve as a bridge to generalize these basic principles when they deal with algebraic expressions and equations in later grades (Carpenter, Levi, Franke, & Zeringue, 2005). For instance, the distributive property (DP) of multiplication over addition becomes a powerful algebraic tool when we consider an expression like “$8x + 4x$.” By DP, $8x + 4x$ equals $(8 + 4)x$ or $12x$. Indeed, it is the DP, which is behind (or justifies) the popular language, used in many school algebra texts, that although one can add or subtract “like terms,” one should not attempt to simplify the addition or subtraction of “unlike terms.” Hence, $8x + 5x$ equals $13x$, but $8x + 5$ cannot be further simplified. The same DP, but thought of in the “reverse direction,” can be called upon to justify statements like $5(2x - 1) = 10x - 5$.

Cai and Knuth (2011) drew attention to the two dominant perspectives in algebra education research. The first is related to a perceived need to develop students’ algebraic reasoning so that they will be able to make strong connections between the arithmetic and algebra. For instance, the recognition, formalization, and use of structures within sets of numbers can involve reasoning about operations and structural properties with respect to the sets of numbers. Further, they can reflect on whether the associative or commutative properties hold for different operations when they are applied to different sets of numbers. The researchers also pointed to the second perspective, which is related to the importance of supporting teachers’ efforts to foster the development of students’ understanding of the properties of operations and how to apply them in different arithmetic and algebraic operations. To be able to foster students’ understanding of the properties of operations and how to apply them, first teachers should have a sound understanding of those properties (Ding, Li & Capraro, 2013). Giving attention to such properties is likely to promote algebraic reasoning. However, very few studies have focused on elementary teachers’ understanding of fundamental number and structural properties and their differences (Ding et al., 2013; Mason, 2008).

In this study, we specifically examined the extent to which PTs learned to notice, to state, and to apply the distributive property and the commutative and associative properties of addition/multiplication for real numbers. Part of the study will be concerned with the extent to which the participating PT’s developing knowledge and understanding of the associative and distributive properties for real numbers helped them not only to formalize the concept of a variable, but also to develop a better understanding of what is traditionally regarded as elementary or early algebra.

Theoretical Framework

The theoretical bases for this study would be a *concept image* (Tall &Vinner, 1981; Vinner & Dreyfus, 1989). The term concept image describes the total cognitive structure that is “associated with a concept, which includes all the mental pictures and associated properties and processes” (Tall & Vinner, 1981, p. 152). Gagné and White (1978) extended this total cognitive structure as being made up of four separable components—verbal knowledge, intellectual skills, imagery, and episodes. The design of the study with this theoretical base enabled salient features of concept images to be identified for each participating PST, both before and after teaching interventions, and with respect to algebraic structure.

An imagined scenario, the sequence and content of the thinking stimulated by the operation "$4 \times (\frac{1}{4} \times 128)$" involved:

- Memory of *verbal information* (concerning PEMDAS);
- An attempt to recall appropriate *skills* ("How do I find the value of 1/4 × 128?");
- Memory of a relevant past *episode* (getting a perfect score on a test);
- *Imagery* (evoking the associative property for multiplication);

For example, when one PT sees 4 × (1/4 × 128), s/he might immediately think the order of operations (PEMDAS) and performs the operation in the parenthesis first, while a second PT might think about multiplying 4 and 1/4 first, whereas a third PT might think: "Ok. the teacher wants to see if I recognize that the associative property of multiplication should be used". It is clear that the second and third ways of thinking are preferable to the first and their concept images differ each other.

In this study, in order to make sense of the qualitative data, particularly the pre and post teaching interview data, we created ordered pairs, which can be used to measure the changes of the participant PSTs' concept images by using Gagné and White's (1978) components of cognitive structure. For a particular component, the extent of evidence was assessed on a three-point scale, 0 (corresponding to no evidence), 1 (some evidence), and 2 (strong evidence).

Methodology

This mixed methods study describes an investigation into the PTs' developing knowledge and understandings of key elementary algebraic concepts. The participants were prospective elementary teachers enrolled in two sections of a mathematics content course (n = 51) at a Midwestern state university and two sections of a mathematics content course (n = 47) at a Southeastern state university. One section at each university received the treatment (an intervention) and the other section served as the control group. In particular, the participating teachers' growth in their knowledge, understanding and use of algebraic concepts associated with the terms "commutative" property, "associative property," and "distributive property," were investigated. The participant completed a paper-and-pencil *Algebra Test* as a pretest and a parallel version of the same test as a posttest in the spring of 2017. The Cronbach alpha reliability of the *Algebra Test* (pre teaching version) was calculated to be .82, and the post teaching version was .83. In addition, 16 students were interviewed at the pre and post teaching stages on a one-to-one basis according to the interview protocol recommended by Newman (1983) and Goldin (1998). The goal of the interviews was to ascertain how the students were thinking about designed task-based, pencil-and-paper instruments. In this study, the following framed research questions were addressed:

1. What did the participating prospective teachers know about each of the commutative property, the associative property, and the distributive property, *before* the intervention lessons took place?

2. What changes in the knowledge and understanding of participating teachers with respect to structure was evident after the intervention period? In addition, what were the Cohen's *d* effect sizes for the treatment and control groups after the interventions?

3. Immediately after the workshops were completed, were there educationally noticeable differences between the concept images of the prospective teachers, with respect to the concept images that the students had before the interventions began.

Intervention

Teaching intervention constitute a series of workshops conducted during six sessions of regular mathematics education content courses, in which the participants explored algebraic structures and number properties. The aim for the workshops was to involve all of the students actively and expressively in the learning process so that they would achieve an understanding not only of the "variables" aspect of statements such as, $(a + b) + c = a + (b + c)$, $(a \times b) \times c = a \times (b \times c)$, and $a \times (b + c) = a \times b + a \times c$, but also of how those properties are vitally important in the development of standard ways of operating in elementary algebra (for solving equations and inequalities, and creating equivalent algebraic expressions).They also provided justification for mental arithmetic calculations (such as finding the value, mentally, of 803 + 798, or finding the value of 25 × (4 × 19), or finding the cost of 11 pens at $1.05 each) and developing the number sense via enhancing students' structural understanding.

Results and Conclusion

To answer our first research question, we asked PTs to describe the commutative property of addition and multiplication, the associative property of addition and multiplication, and the distributive property of multiplication over addition in their own words on the pretest. Most of the PTs who were in the treatment group either stated that they did not know about the property being asked or they did not correctly describe the property. The percentages of PTs who did not know or who did not provide a correct description of the commutative, associative and distributive properties were 74%, 93%, and 63%, respectively. Although many students stated that they did not know the property being asked, some provided a description. Examples for the incorrect descriptions included but not limited to:

CP of addition and multiplication: "Move numbers around to solve the equation", "Figuring out one side's total to help you figure the other side's total", etc.

AP of addition and multiplication: "Write the set of number in any order and still get correct solution", "The associative property is when you either use multiplication or addition to get the same value. For example 2+2= 4 and 2x2=4", etc.

DP of multiplication over addition: "When you multiply a number into a parenthesis, you multiply every number in the parenthesis by that number, Ex: 2 (4x5) = (8x10)", "You will need to use both variables in parts to get the total sum. For example (8x4)(3x5) = (8x3)+(8x5)+(4x3)+(4x5) ", etc.

To answer our second research question, we examined the differences in the Algebra Test scores between the treatment and control groups both before and after the interventions. Before the intervention, there was no significant difference between the treatment and control groups. (t=.061, df=89, p=.952). However, there was a statistically significant difference in their Algebra Test scores between the treatment and control groups after the intervention (t=10.732, df=72.6, p<.001). The PTs in the treatment group used the CP, AP, and DP to solve the problems on the Algebra Test significantly more than the PTs in the control group. Tables 1 and 2 display the descriptive statistics at the pre and posttest level.

Table 1

Descriptive statistics at the pre teaching test level

	Group	*N*	Mean	Std. Deviation	Std. Error Mean
Pre teaching	Treatment	52	3.06	2.704	.375
	Control	39	3.03	2.170	.348

Table 2

Descriptive Statistics at the post teaching test level

	Group	*N*	Mean	Std. Deviation	Std. Error Mean
Post teaching	Treatment	52	12.29	5.259	.729
	Control	37	3.57	2.180	.358

According to Cohen (1988), effect sizes above 0.8 can be regarded as "large." Based on pre and post teaching data from the treatment and control group, the effect of the intervention groups' performance was very large (Cohen's *d* effect size was 2.11). The effect size was very large, and it seemed to be the case that the study's intervention workshops were effective.

The items on the Algebra Test did not include just any randomly-ordered and randomly-chosen symbols. They were designed to invite the applications of the CP, the AP or the DP. (See appendix for the sample items on the Algebra Test). On the pretest, most PTs did not attempt to use the properties, rather they followed the PEMDAS rule. For example, the typical solution on the pre-test to the problem "What would be a quick method of finding the value of 7 x 97 + 7 x 3? What is the property, which allows you to use that quick method?" was "Multiply 7 by 97 and 7 by 3, then add them". At the posttest, the typical solution to a parallel problem "8 x 96 + 8 x 4" was "Change it to 8 (96 + 4) to get 8 x 100 = 800". Most of the PTs who applied this solution also provided the name of the property that allowed them to use this method.

An example related to the AP of multiplication was the problem: " Without using a calculator find the value of $(72\times 5)\times 2$ ". On the pretest, most PTs followed the PEMDAS rule and multiplied the numbers inside the parenthesis first. On the posttest, most PTs associated 5 and 2 first, and therefore computed the answer efficiently.

An example related to the AP and the CP of addition was the problem "… calculate the value of *x* in this equation "463 + 1999 = 2000 + *x*" in your head…" The typical solution provided on the pretest to this problem was "I would add 463 and 1999 and get a number. Then I would subtract 2000 from that and that number would equal *x*". At the posttest, many PTs used the commutative and associative properties to solve a parallel question, " $563+999=1000+x$ " The typical solution was "I would give 1 from 563 to 999 making it 562 + 1000 to get 1562 = 1000 +

x and I would know the answer has to be 562". Figure 1 shows the AP and the CP in PTs' solution.

$$563 + 999 = 1000 + x$$
$$(562 + 1) + 999 = 1000 + x$$
$$562 + (1 + 999) = 1000 + x \quad (\textit{Associative property of addition})$$
$$562 + 1000 = 1000 + x$$
$$1000 + 562 = 1000 + x \qquad (\textit{Commutative property of addition})$$

Figure 1: Sample item for the associative and commutative property of addition.

To answer our third research question, we asked PTs to describe the CP of addition and multiplication, the AP of addition and multiplication, and the DP of multiplication over addition in their own words at the posttest. Most of the PTs in the treatment group provided a correct written description and an example of the properties. The percentages of PTs who did not provide a correct description of the CP, AP, and DP at the posttest were 24%, 28%, and 15%, respectively.

The initial analyses of quantitative data revealed that most of the participating PTs' concept images with respect to structural properties changed in educationally significant ways. Many PTs took the opportunity to construct their own concept images; they were likely to remember much of what they had learned.

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Appendix

Algebra Test Sample Questions

1. A really important property for numbers and for algebra is called the commutative property for multiplication and addition. Describe this property in your own words?
2. A really important property for numbers and for algebra is called the *associative property for multiplication and addition*. Describe this property in your own words?
3. A really important property for numbers and for algebra is called *the distributive property multiplication over addition*. Describe this property in your own words?
4. Suppose you were asked to calculate the value of 940 + (60 + 403) in your head (without writing anything down, or using a calculator. How would you do it, and which property would you be using?
5. What would be a quick method of finding the value of $8 \times 96 + 8 \times 4$ without using a calculator? What is the property which allows you to use that quick method?
6. What would be a quick method of finding the value of $64 \times (\frac{1}{32} \times 160)$ without using a calculator?

ELEMENTARY PRESERVICE TEACHERS' LEARNING TO EXPLAIN THE INVERT AND MULTIPLY ALGORITHM

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While elementary preservice teachers (PTs) may remember and apply the invert and multiply algorithm for fraction division procedurally, they rarely are able to explain its meaning using a scenario. This study examines PTs' responses to an assessment task that requires them to explain the invert and multiply algorithm for fraction division using a context of their creation and its related ratio table. Their responses to the assessment task reveal the key concepts that contribute to PTs' conceptualization of the algorithm through their explanations of the "invert and multiply" algorithm using fair sharing contexts.

The mathematical practice of modeling requires understanding the relationship between mathematical symbolizations and the contexts that they represent. Most research focuses on the process of identifying quantities in practical situations and mapping their relationships into types of representations such as diagrams, tables, graphs, and formulas to facilitate analysis and to formulate answers to questions that arise (CCSS, 2010, Gravemeijer, 1994). A deep understanding of mathematical modeling should require not only being able to symbolize quantities in a context mathematically, but also to reverse the process by providing contextual models for mathematical symbols and processes (Piaget, 1963). Teachers in particular need to understand how to model mathematical symbols and processes using contexts. One such mathematical process is the invert and multiply algorithm for fraction division.

Ma's (1999) analysis of U. S. and Chinese practicing teachers' explanations of fraction division included explorations of both teachers' procedural reasoning and their ability to create contexts for representing fraction division. Teachers were asked to solve and explain their solution to a fraction division problem and then to provide a context to model the solution process for the problem. Roughly half of the U. S. teachers interviewed provided a successful solution process, typically either the common denominator algorithm or the invert and multiply algorithm to calculate the solution. Most of the successful U. S. teachers used the invert and multiply algorithm. However, only one of the successful teachers produced a context that appropriately represented one of the two algorithms. The reported explanation approximated the common denominator algorithm although there was no indication of which algorithm the teacher used for calculation. Ma's research suggests that a contributing factor to PTs' inability to conceptualize the invert and multiply algorithm may be a lack of knowledge about contexts that could support meaningful explanations of the algorithm.

Empson and Levi (2011) observed that children who had been encouraged to reason relationally when solving partitive (fair sharing) division problems, used processes could be interpreted as using an invert and multiply strategy. To provide PTs with a conceptual understanding of the invert and multiply algorithm for fraction division Gregg and Gregg (2007) designed an instructional sequence using a context that promoted a fair sharing or partitive division model for fraction division. The context was the putting the same amount of cake into some number of containers and asking how much of a cake would be placed in one container. Initial problems begin whole numbers of containers and partial amounts of cake and eventually move to partial containers. In other words, "If ¾ cake fits into ½ a container, how much cake would fit in one whole container?" Empson and Levi called these Partial Groups or Partitive division problems and describe the problem structure as having a known amount of group(s), a known total amount, and asking for an unknown amount per group. The work of Empson and Levi and Gregg and Gregg therefore suggested that fair sharing or partitive division contexts are appropriate for supporting the explanation of the invert and multiply algorithm.

Vergnaud (1988) observed that all multiplicative structures such as partitive (fair sharing) division are equivalent to proportions. The partitive division model described both by Empson and Levi (2011) and Gregg and Gregg (2007) is isomorphic to solving a proportion of the type $a/b = x/1$ such that a and b are known quantities and x is the quantity of interest. Subsequent to Gregg and Gregg's (2007) development of instructional sequences for partitive division of fractions, the instructor/researcher (first author) revised and extended their instructional sequences to use varied fair sharing contexts and a ratio table representation to promote PTs' development of a contextual model to explain the invert and multiply algorithm for fraction division. An assessment tool that reversed the process that PTs experienced in the instructional setting for the course was designed to capture PTs' capacity to explain the invert and multiply algorithm using a scenario. This study focuses on what is learned about PTs' conceptualizations of the invert and multiply algorithm through the use of the assessment tool.

Methodology

The setting for the study was a university in the Pacific Northwest. As part of a first content course for elementary teachers, PTs were introduced to solving fair sharing problems with fractions as a model for conceptualizing the "invert and multiply" algorithm for fraction division. PTs received a homework assignment similar to the assessment that required them to explain the invert and multiply algorithm using a context. They were encouraged to work in study groups and with the instructor on similar tasks prior to the assessment. The assessment consisted of three items one of which was designed to capture PTs' conceptualization of the invert and multiply algorithm. The assessment item from which the data is collected is composed of a fraction division problem such as 7/5 ÷ 2/3 = ? followed by the prompts:

- Number sentence/context. Write a number sentence for the problem using units from an appropriate scenario for the "invert and multiply" algorithm.
- Ratio table illustration. Create a labeled ratio table to illustrate the solution using the scenario you chose.
- Written explanation. Explain the "invert and multiply" algorithm using the scenario from your solution.

The data set consisted of all responses to the assessment item from all PTs who participated in the content course for elementary teachers—105 PTs total for two sequential quarters. For the assessment, PTs worked on their own in one sitting and submitted their work to the instructor upon completion. Although a set amount of time was not allotted to the assessment, the maximum time taken for the whole assessment was typically one hour, one third of which was attributable to the item reported here.

The theoretical foundation for the data collection and analysis is a constructivist frame as articulated by Von Glasersfeld (1995) and grounded theory as articulated by Strauss and Corbin (2015). The constructivist frame supports the use of PTs' assessment responses for providing evidence of PTs' conceptualizations of the invert and multiply algorithm. Although the researcher cannot watch how a PT builds her/his concepts, the concepts can be investigated by examining the words, symbols and other tools PTs use in their responses to describe intended quantities and their relationships and by examining their responses for elements the associated concept should include to reflect the quantities and their relationships adequately.

Grounded theory supports the use of a constant comparative method to code PTs' responses to the assessment item and to generate cases from the analysis of the coding to develop a theory of the contribution of proportional reasoning (equivalent ratios) as represented by fair sharing problems and ratio tables to conceptualization of the "invert and multiply algorithm" for fraction division. PTs' responses to the number sentence/context task, solution illustration task, and written explanation task were compared and contrasted separately and together for evidence of contributing evidence for both understanding a contextual representation of division and how the contextual representation could be used to explain the "invert and multiply" algorithm for fraction division. For the number sentence/context task, responses were categorized for the types of scenarios PTs created: fair sharing (partitive), measurement (quotitive), multiplication or other. For the solution illustration task, PTs' responses were categorized for whether or not the illustrated solution processes using ratio tables or pictures could represent proportional reasoning with the quantities in the scenario from the number sentence/context task and if not what other categories of reasoning were evident. For written explanation task, PTs' responses were categorized for: (a) the focus of the explanation such as context, process, or procedure and (b) the correspondence between the representation of the solution in ratio table and the written explanation of the solution—i.e., could the process described in the explanation be the same process illustrated in the ratio table. Additionally, the written explanations were categorized for evidence of conceptualization of the algorithm through answers to the following questions that relate the responses across the three tasks:

- Does the written explanation use the scenario from the number sentence/context and the ratio table to explain each part of the solution process illustrated?
- Is the solution process in the ratio table reasonable and complete for the context created and does it align with proportional reasoning?
- Does the written explanation also explicitly use the solution process described with the scenario in the ratio table to make sense of the associated process within the "invert and multiply" algorithm?
- Does the scenario in the number sentence/context contribute to or inhibit the written explanation?

Important to understanding and validating the data collection and analysis is the acknowledgement that the researcher and instructor of the course are the same. The researcher/instructor makes a claim to have expertise in analyzing students' productions for representations of knowledge of the topic of instruction because of her instructional experience in teaching and assessing students' reasoning on the topic using the curricular materials based on the research of Empson and Levi (2010) or created by Gregg and Gregg (2007). Trustworthiness is also supported through the triangulation of the data through using multiple types of questions that comprise the data, the collection of the data across multiple groups and times and to the close attention paid to what the PTs' write in their explanations and how they write them (Strauss and Corbin, 1998).

Cases

The categories generated for conceptualization of the algorithm were subsequently analyzed to create the following cases. The cases have been compared and ordered on the category of conceptualization of the algorithm from least representative of conceptualization to most representative of conceptualization.

No Meaning for Math Symbol Sentences Yet

PTs in this category might rewrite the division problem given (7/5 ÷ 2/3 = ?) as a multiplication sentence for (1) such as "7/5 brownie x 2/3 serving = ? brownie in one pan" or reverse the quantities in the division problem. The process described in the ratio table that they create may represent an appropriate scenario for the given division problem or the reversed division problem. Their explanation may follow the process described in the ratio table, but it could lack appropriate details, use inconsistent units, or refer to unrelated concepts. For example:

> "I have 7/5 brownie and 2/3 of a brownie is a serving. I am trying to find how many brownies per pan. I start with 2/3 and multiply by ½ to get the amount for 1/3, which is 7/10. Then when I multiply what happens is I am making the 7/5 into 1/10 because I cut the 1/5 into 1/10 to get the amount with the same size pieces, which is 1/10's. I must find this and then I multiply to find the amount of brownies per serving."

The evidence suggests a procedural process for the solution and explanation but little or no understanding of how to use a context to give a symbol sentence meaning.

Lack of Proportional Reasoning

PTs in this category often have difficulty solving the problem using a ratio table. They may add amounts to "find" solutions or divide 2/3 by 2 and multiply 7/5 by 2 as their initial step in the process for solving 7/5 ÷ 2/3 = ?. Their explanations may revert to descriptions of the algorithm even if they begin with an appropriate scenario such as "7/5 cups of flour are 2/3 recipe. How much flour is used for one recipe?"

> "When you invert and multiply you are actually using a ratio to solve the equation. When you invert you are trying to find the scalar factor for the ratio table. Inverted means you are switching the 2/3 into 3/2 and then multiplying the 7/5 by the 3/2."

Although the PT claims ratios were used to solve the problem, the ratio table showed 63/10 cups of flour were used for 1/3 recipe. The final ratio in the table 21/10 cups for 1 recipe suggests the PT used a different process to find a solution.

Misunderstanding Multi-digit Fractions

A number of PTs seem to have difficulty understanding multi-digit fractions. They may have created an appropriate scenario for $7/5 \div 2/3 = ?$ such as "7/5 cups of sugar is in 2/3 of a recipe. How many cups of sugar is in one recipe?" And in their ratio table they know to multiply 7/5 cups and 2/3 recipes by the same amount to find how much cups of sugar is in 1/3 recipe. However, they think 1/3 recipe is 1/3 of 2/3 recipe and multiply 7/5 cups by 1/3 as well to get 7/15 cups for 1/3 recipe. They often report a result of 21/15 for the solution to $7/5 \div 2/3$ and their explanations describe the process from the ratio table using the scenario.

Confusion from Misaligned Scenarios

Some PTs create a measurement division scenario rather than a fair sharing division scenario. They may be able to solve and illustrate the problem appropriately in the ratio table, and they may be able to explain the process meaningfully using the scenario, but they become confused when connecting their explanation to the "invert and multiply" algorithm. For example the following explains 7/5 cups ÷ 2/3 cups/recipe = ? recipe:

> "When you invert and multiply you are really taking the 2/3 cups and dividing that into 2 pieces to get 1/3 cups and you have to divide the 1 recipe by 2 to get ½ recipe. You then divide the 1/3 by 5 and the ½ by 5 to get 1/15 cups for 1/10 recipe. Then you multiply by 7 to get 7/5 cups for 21/10 recipe. Inverted means to flip the fraction upside down… This made more sense with other numbers."

In the section covered by the ellipsis, the PT appropriately described what each of the elements of 7/5 x 3/2 represented within the scenario. However, the PT attributed the complexity in the explanation of the algorithm to the "numbers" rather than the choice of scenario.

Process Based Explanations

Some PTs appropriately solve a division problem using a context, but their explanations focus on only one of the two units involved. Usually, it is the unit that results in 1 whole at the end of the process. For example:

> "Since I have 2/3 serving and I want to find 1/3 serving I multiply by ½ and this is where the 2 in 2/3 flips to the bottom and is multiplied. And then to find 1 serving we multiply the 1/3 by 3 in order to get 3/3 which is 1."

These explanations often feature connections to the "invert and multiply" algorithm and suggest conceptualization of the algorithm. However, they focus on the solution process and lack the supportive details that provide meaning.

Solving Without Connecting to Algorithm

Many PTs are able to appropriately solve and meaningfully explain a division problem but may not explicitly connect the process to the "invert and multiply" algorithm or may have unclear or inaccurate descriptions of how the process connects to the algorithm. In the following example for the scenario 7/5 cups ÷ 2/3 serving = ? cups/serving, the underlined sentence is unclear in its meaning for either the scenario or the algorithm although the explanation of the solution process using the scenario is meaningful and clear.

> "We know we have 7/5 cups for 2/3 serving. Our goal is to find how many cups go into 1 serving. <u>First if we multiply by ½ we will find how many total pieces we will have</u>. Then if we multiply by 3 we will see how many pieces of the whole we actually used. In our case we had 7/5 cups x ½ = 7/10 cups which is 1/3 of a serving. Then we multiply 3 by our servings to get 1 whole serving and 7/10 x 3 = 21 cups for 1 serving."

"Invert and Multiply" Contextual Explainer

The following explanation is an exemplar of PTs' who were categorized as having conceptualized the algorithm. The PT represented 7/5 ÷ 2/3 = ? by the scenario "2/3 serving is 7/5 cookies. How much cookie is in a full serving?" and explained:

> "My first step for finding number of cookies in one serving was to go from 2/3 of a serving to 1/3 of a serving. In order to do this I had to multiply 2/3 by ½ since 1/3 is half of 2/3. I simultaneously multiplied 7/5 by ½ because of the scalar relation. This is when the numerator gets flipped and multiplied in 2/3 serving. Once I find 1/3 of a serving, all I have to do is multiply 1/3 serving by 3 to get 1 whole serving. I also simultaneously multiply 7/10 by 3 because of the scalar relation. This is when the denominator gets flipped and multiplied in 2/3 serving."

Cross Analysis and Conclusion

The cases generated from PTs' responses to the assessment item provide a range of the PTs' conceptualizations of the "invert and multiply" algorithm. They reveal key features of understanding mathematical processes such as the "invert and multiply" algorithm. The fraction and proportional reasoning concepts that PTs develop to be able to explain the invert and multiply algorithm using fair sharing contexts include the following:

- The multiplicative relationship between a multi-unit fraction and its related unit fraction,

- The multiplicative relationship between a unit fraction and its related whole,
- Coordination of units in a proportional relationship,
- Multiplicative relationships between equivalent ratios,
- Contextual representations of proportions,
- Fair sharing situations represent proportions, and
- Conceptualization of the "invert and multiply" algorithm requires proportional reasoning.

The use of scenarios to explain the "invert and multiply" algorithm provides a rich venue for PTs to develop proportional reasoning and to revisit their understanding of fraction relationships as well an opportunity to prepare to teach the algorithm meaningful. However, it does not represent the only method for PTs' to explore the meaning of the "invert and multiply" algorithm. Other explanations of the "invert and multiply" algorithm should be explored and connected to the use of scenarios. Additional research should explore whether the developed here cases may represent a learning progression for the development of key concepts that contribute to the conceptualization of the algorithm.

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BUILDING FLUENCY IN AN ARITHMETIC COURSE FOR PRESERVICE ELEMENTARY TEACHERS

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This study investigates the use of a fluency-building routine in an arithmetic course for elementary teachers. This paper highlights strategies used to develop fluency with whole number operations; fraction, decimal, and percentage comparisons and operations; and estimation. These strategies focus on building multiple ways of arriving at an unknown answer by working from what students already know and understand. This paper also details the methods used to collect data that describe student experiences with this routine.

Rationale

Seaman and Szydlik (2007) explain, "Many preservice teachers begin their education programs unable to perform basic computations or to explain fundamental mathematical ideas" (p. 178). Given that preservice teachers are eventually expected to foster procedural fluency with their own students, the argument by Seaman and Szydlik presents a multi-faceted challenge for mathematics teacher educators. Not only must mathematics teacher educators foster fluency with preservice teachers, which is one of the five strands of mathematical proficiency (Ball, et al., 2005; National Council of Teachers of Mathematics [NCTM], 2014; National Research Council [NRC], 2001), they also must help preservice teachers develop high-leverage teaching practices that enable their students to build procedural fluency from conceptual understanding (NCTM, 2014). Consequently, building procedural fluency must be a point of emphasis in preservice teacher education (Conference Board of the Mathematical Sciences [CBMS], 2012; NCTM, 2014; Seaman & Szydlik, 2007).

Being fluent means choosing methods or strategies appropriate for the given problem and efficiently producing and explaining accurate answers (NCTM, 2014). Some of the necessary components for fluency are: automatic recall of addition and multiplication combinations of integers 0 through 10; addition, subtraction, multiplication, and division of multidigit numbers both through the use of mental math strategies and standard algorithms; estimation; and the application of procedures with flexibility, understanding, and accuracy (Ball et al., 2005; NRC, 2001). Building fluency with preservice elementary candidates requires regular entrenchment in the culture and habits of mathematical thinking so that preservice teachers realize that mathematics is much more than memorizing and applying formulas and procedures (CBMS, 2012; Seaman & Szydlik, 2007).

Instructional Methodology

This study took place in an arithmetic course for elementary teachers at a public, open enrollment university. While most of this course focuses on equipping students with conceptual understanding of pre-established rules, operational meaning, appropriate representations of mathematical ideas, and authentic problem-solving opportunities, one long-standing course requirement has been that preservice teachers be able to demonstrate procedural fluency with K-6 arithmetic. Instructors assess students' fluency by administering a series of three computational fluency exams. To support students in achieving procedural fluency, the course utilizes a daily fluency routine, which is described in detail below.

Establishing a Routine

Each student keeps a mathematical fluency journal where she documents all the work done to build procedural fluency. Journals are graded for completeness and growth achieved with fluency. During the first week of the course, students take a computational fluency exam, which is comprised of 25 questions that test computational fluency for addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals. Some of the problems test facility with associated algorithms, and some of the problems provide opportunities for students to use mental math strategies. Students have three opportunities during the semester to pass the exam with an 80% or better, which is a requirement for passing the course.

Instructors lead discussions with students about what procedural fluency is and why it is important for preservice teachers and their future students. To make this point, students are asked to think about several ways to get from campus to their house, and then they are asked to consider, "Why would you choose one of the routes over another?" The students cite reasons such as time, weather, traffic, and after school errands as reasons. The analogy is completed by explaining that mathematical fluency is the ability to look at a problem, think of several ways to approach a problem, and choose a particular method because it is best suited to the problem and the knowledge of the solver.

Finally, during the first week, students complete the Mathematical Fluency Indicator at home. This indicator gives students targeted feedback on which areas of fluency they need to concentrate their efforts. The indicator is divided into four sections: addition, multiplication, fraction to decimal conversions, and estimation. For each section, students record in their journals the number of correct answers and the completion time.

Daily Routine

Upon entering the classroom each day, each student picks out a fluency warm-up activity to work on until one minute into class. Students record their work on the chosen activity in their journals. When class begins, the instructor leads a routine in which students share sticky points and breakthroughs. Sticky points are particular problems that proved to be challenging for a student, and the rest of the class is asked to think of possible ways to solve the problem. Often ideas are shared through a number talk routine (Humphreys & Parker, 2015). Breakthroughs are new ideas or ways of thinking about an area of fluency, and these are shared to encourage other students. Later in the course, students take turns leading these discussions, which allows them to practice generating, recording, and debriefing student responses. The fluency warm-up activities and subsequent discussion takes between two and ten minutes at the start of each class period. Students record in their journals new strategies they learned from these discussions. The following exercises are explained in the order they are introduced during the course.

Build it! In this activity, students sort multiplication facts according to whether or not the fact is recalled automatically. For any fact that is not, the students work to build the fact from a related fact they do know. For example, when determining the product of 8×7 students might view the problem as $8(5+2)$, $7(5+3)$, or $(7\times4)2$.

30-second challenge. In the 30-second challenge books (Lock, 2010), students are given an initial number and then asked to perform a series of operations on the number using mental math. The challenges are sorted into beginner, intermediate, and advanced puzzles. Some examples of sticky points in this activity include $\frac{3}{5}$ of 600, $119 \div 7$, and 75% of 300.

24. Using a standard deck of cards, students flip over four cards and try to be the first to make the cards equal 24 by using every card exactly once with some combination of operations. All face cards are worth 10, and aces are worth either 1 or 11. Students must also write their solution as one expression to win a point. For example, students might flip over 4, 8, K, and Q. One possible solution would be $\{[(8-4)(10\div10)]!\}$.

Fraction wall. Each partner begins with an empty fraction wall as shown in Figure 1, and takes a turn rolling a numerator die (1, 1, 2, 2, 3, 4) and denominator die ($\frac{}{2}, \frac{}{3}, \frac{}{4}, \frac{}{6}, \frac{}{8}, \frac{}{12}$).

After the roll, the student is tasked with shading an amount on the wall equivalent to the result of the roll. The student may shade any area equivalent to the rolled amount, but has to shade the entire area to be able to complete the turn.

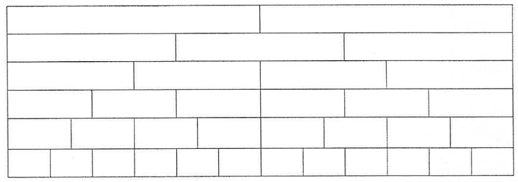

Figure 1. Fraction wall

Fraction war. A stack of fraction cards is split between two partners and held face down. Each partner flips over a fraction to compare, and the person with the larger fraction wins both cards. To win the cards, the student must explain how she knows her fraction is larger. As a variation, students take turns determining which fraction is closer to 0, 1, or ½. Later in the course, these cards become fraction estimation cards. In this case, two fraction cards are flipped over, and students determine whether the sum (or the quotient) will be more or less than one.

Estimation station. This warm-up game has two stacks of cards: the first features whole numbers, fractions, decimals, and percentages, and the second features an operation ($of, +, -, \times, \div$) followed by a number. In pairs, students turn over one card from each stack. Each partner is then given an agreed upon amount of time (e.g., 15 or 30 seconds) to lock in an estimate. After each student has locked in an estimate, the students may find the exact answer using a calculator to determine which estimate was closest. At this time the students also compare how they estimated. Some of the problems produced by this activity include $\frac{11}{6}$ of 2845, $14.6 + 3\frac{2}{5}$, and $\frac{7}{12} \div 0.16$.

Research Methodology

To understand how students grow with procedural fluency in the areas of addition and multiplication facts with integers 0–10, fraction to decimal conversions, and estimation of addition, subtraction, multiplication, and division problems involving whole numbers, decimals, fractions, and percentages, data from 25 preservice elementary students' fluency journals was collected in the spring 2017 semester. The data includes students' self-reported time and accuracy for each of the four categories of the Mathematical Fluency Indicator, taken during the first and last weeks of the semester. Additionally, the data includes the first and last entries in the journal in which students reflected on their experiences taking the Mathematical Fluency

Indicator. Similar data collection is occurring in two sections of the course during the fall 2017 semester; analysis of that data will be reported at the 2018 RCML Annual Conference.

Findings

Mathematical Fluency Indicator

A paired-samples *t*-test was conducted to compare student time and accuracy on each of the four Mathematical Fluency Indicator categories taken at the beginning and end of the semester (see Table 1 and 2). This data shows statistically significant improvement in both accuracy and speed in all four categories.

Table 1
Pre- and Post-Comparison of Proportion Correct on the Mathematical Fluency Indicator

			Paired Differences				
	Pre	Post	Mean	Std. Dev.	*t*	df	Sig. (1-tail)
Addition	0.988	0.997	-0.008	0.024	-1.735	23	0.0480
Multiplication	0.939	0.974	-0.035	0.056	-3.102	23	0.0025
Fraction-Decimal	0.711	0.918	-0.207	0.222	-4.378	21	0.0001
Estimation	0.682	0.852	-0.170	0.161	-5.077	22	0.0000

Table 2
Pre- and Post-Comparison of Time in Seconds on the Mathematical Fluency Indicator

			Paired Differences				
	Pre	Post	Mean	Std. Dev.	*t*	df	Sig. (1-tail)
Addition	100.167	76.583	23.583	21.472	5.381	23	0.0000
Multiplication	158.667	98.667	60.000	50.53	5.817	23	0.0000
Fraction-Decimal	337.714	145.095	192.619	141.997	6.216	20	0.0000
Estimation	433.478	294.565	138.913	107.651	6.189	22	0.0000

Beginning of the Semester Journal Entry Themes

Many of the students struggled with the indicator at the beginning of the semester. This was most pronounced in the areas of fraction to decimal conversions and estimation.

Lost skills. A majority of the students expressed surprise for how much they struggled and attributed this struggle to forgetting mathematics they once knew. Many students cited the amount of time that had elapsed since they first learned these ideas. One student explained, “Taking the math fluency indicator was interesting because I have not done this kind of math for quite a while, so I am pretty rusty.” Interestingly, some students did not even remember learning these topics in school. Some students pointed to calculator use as a reason they had forgotten how to do this mathematics.

Not good at this. Many students wrote about how this indicator confirmed the areas of chronic mathematics struggle. These areas were estimation, fractions, and "times tables." The students used phrases like "I'm terrible at…" "I've always had a hard time with…" and "I'm bad at…." One student concluded, "I'm not very good with fluency."

Anxiety. Many of the students wrote about how the indicator triggered their long-standing mathematics and testing anxieties. One student wrote, "Taking these fluency indicators really freaked me out. I haven't felt this dumb in a very long time." Many of the students also communicated being afraid of not being able to gain fluency during the course because of past failures in mathematics. Several students pointed to the timed aspect of the test as a reason they were freaked out and did not do well on the assessment.

End of the Semester Journal Entry Themes

All of the students performed better on the indicator at the end of the semester.

Growth surprise. The most common theme in the last entries was student surprise at how they grew with fluency. Many of the students reflected on the way they used to do mathematics in "the longest and most arduous ways of solving" and marveled at how their present thinking differed. Students who at the beginning of the semester thought they were just "rusty" with mathematical fluency admitted that they had never truly possessed mathematical fluency before, and only now understood what true fluency meant. Just as Seaman and Szydlik (2007) established, these students confirmed that they had previously seen mathematics as a set of rules and procedures you use to find an arbitrary correct answer. One student explained that the class warm up exercises and thinking strategies "have rewired the way I think." Another student rejoiced at her new found tool of estimation because she "no longer goes into a problem, blindly, wondering what the answer will be." Another student explained that at the beginning of the course her "go to tool was a calculator, not my brain," by the end of the course she did "real life math" without a calculator. One student explained, "I can now build from the things I know when I forget other things." The students frequently used words such as understanding, active thinking, reasoning, and common sense to explain how their approach to mathematics differed by the end of the course; "This class has raked my brain over hot coals, but here at the end, I now feel I have a true understanding of basic math."

Growth mindset shifts. While many of the beginning journal entries bemoaned innate struggles with mathematics, so many of the final entries attributed newly gained fluency to hard work and effort. This was a common sentiment by the end of the course, "Some things are still challenging, but I know that I can get it if I keep working at it." One student explained how she used to see mental math as something used only by very smart people, but by the end of the course she could "fully depend on myself and my own knowledge to solve problems." Another student even said he "really looked forward to finding problems that would stump me" because these presented an opportunity to gain a new way of thinking.

All aspects of the course. The students pointed to many reasons for their increase in fluency, including parts of the fluency routine (e.g., warm-up exercises and sticky point discussions), lessons about developing thinking strategies, understanding standard algorithms, using manipulatives, progressing through the concrete→pictorial→abstract sequence, and exploring multiple solution strategies. This connects to mathematics education literature about how procedural fluency, conceptual understanding, multiple representations, and mathematical discourse are interrelated (Ball et al., 2005; NCTM, 2014; NRC, 2001).

Further Study and Implication

The fluency routine created in this mathematics for elementary teacher course and described here connects to the recommendation by Seaman and Szydlik (2007) that preservice teachers be immersed in the culture of mathematical thinking in order to gain necessary proficiency. In fact, one student wrote in her last journal entry, "This is really cool to see my progression and how much better of a mathematician I've become." This routine led to improvement in both accuracy and speed in the areas of addition, multiplication, fraction to decimal conversions, and estimation. Furthermore, the students gained fluency with addition, subtraction, multiplication, and division of whole numbers, fractions, decimals, and percentages as shown on the course fluency exam.

Some changes were made to the fluency routine after this initial data collection. The Estimation Station was developed to help students grow more with estimation. Also, the instructors spent more time explaining that while the Mathematical Fluency Indicator does measure both accuracy and time, time is only included to establish a baseline of completion time for each individual. This baseline helps the instructors and the students see fluency growth throughout the semester. As this fluency routine continues to be researched and refined, the instructors hope to provide a model for use in other content courses for preservice elementary teachers. The researchers look forward to presenting their analyses of subsequent data at the 2018 RCML conference.

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DECIMALS AND FRACTIONS – PRESERVICE TEACHERS' CONCEPTIONS OF THEIR DENSITY

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This qualitative study investigated preservice elementary school teachers' conceptions about the density of the set of decimals and the set of fractions. Thirty-six participants were asked to find a decimal or fraction between two given decimals or fractions and to share their reasoning(s) while answering the four purposefully chosen questions. On average, each question was answered correctly by 67% of the participants. The data suggest that the ability or ease with which participants identify a decimal or fraction between two given decimals or fractions depends on the nature of the numbers. Implications for teaching and assessments are discussed.

Introduction

Teacher Knowledge and Rationale of Study

According to Ball, Hill, and Bass (2005), the quality of mathematics teaching depends on teachers' content knowledge. This is knowledge that can be categorized as being either (a) subject matter (b) pedagogical or (c) curricular knowledge (Shulman, 1986). Shulman points out that strong subject matter content knowledge allows the teacher to go beyond understanding that something is so by understanding why it is so. Researchers (e.g., Ma, 1999; Schoenfeld & Kilpatrick, 2008) have argued that effective instruction, which can in part be demonstrated by teachers providing sound explanations of mathematical concepts, requires teachers to thoroughly understand the content they teach. "To assume that the content of first-grade mathematics is something any adult understands is to doom school mathematics to a continuation of the dull, rule-based curriculum that is so widely criticized" (Ball, 1988, p. 23). Post, Harel, Behr, and Lesh (1988) examined middle school teachers' understanding of rational number concepts and observed that only a few of the teachers who solved the problems correctly were able to give satisfactory explanations of their solutions. The researchers also found out that some of the misunderstandings found in students were also present in teachers.

Studies on inservice and preservice teachers' content knowledge abound, an indication of researchers' view on the importance of teachers' content knowledge. Among the topics investigated through multitudes of studies, are teachers' conceptions about and operations with rational numbers and place value. The current study is concerned with preservice teachers' understandings of density and ordering of fractions and decimals. We embarked on this study with a view that in addition to being able to solve problems, teachers should be able reason mathematically, a view shared by other researchers (e.g., Rathouz, 2009) and that preparing teachers to teach classrooms where children learn to justify their thinking requires that teachers be exposed to an environment where they inquire, communicate, collaborate and reason (Rathouz). Another motivation for carrying-out this study is that even though according to Kastberg and Morton (2014) very few articles focusing on preservice teachers' knowledge of decimals were published since the late 1990's, a common finding in those few studies is the importance of place value in understanding and applying decimals. Investigating conceptions about density and ordering of fractions and decimals in this study was structured so as to understand both the ability to solve the problems and offer plausible explanations.

Literature Review

Erroneous rules applied by students when trying to decide which of two decimals is larger can be classified as longer-is-large or shorter-is-larger. In some cases a zero in the tenths place elicits a response suggesting students think that if a number has a zero in the tenths places, then it is always smaller (Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvald & Leonard, 1985).

In a study involving preservice teachers, more than half of the participants failed to arrange 0.606, 0.0666, 0.6, 0.66 and 0.060 in increasing order (Putt, 1995). In about three-quarters of the incorrect responses, 0.6 was selected as the largest, an indication of the existence of the shorter-is-larger misconception, an indication of an early understanding of place value that says as one moves to the right of the decimal point there is a decrease in positional value (Kastberg & Morton, 2014). A study by Stacey et al. (2001) revealed difficulties comparing decimals with the number zero and some participants who exhibited the shorter-is-longer misconception. Widjaja, Stacey and Steinle (2008) asked preservice teachers to figure out how many decimals are between 2.18 and 2.19 and between 0.899 and 0.90. About half of the 140 participants and about one-fourth did not successfully answer the questions on the pretest and respectively, an indication of difficulties with the idea of density of decimals.

Post, Harel, Behr, and Lesh, (1988) investigated elementary school teachers' ability to order fractions and decimals. On average, participants from one site correctly answered 65.3% and 54.2% of the items in each category. The item with the highest level of success (76%) was writing a fraction between 7/8 and 1 while the item with the lowest success rate (50%) was ordering the fractions 5/8, 3/10, 3/5, 1/4, 2/3, 1/2 from least to greatest. Only 58.7% of the participants were able correctly order .3, .3157, .32, .316 from smallest to largest while 49.7% correctly wrote a decimal between 3.1 and 3.11. Among the researchers concerns, were teachers not knowing enough mathematics as evidenced by the failure of a lot of them to answer questions involving fractions and decimals and the inability to explain solutions in pedagogically acceptable ways by a majority of those who correctly solved problems.

This study investigated the ability to find a decimal (fraction) between two given decimals (fractions) and reasoning provided by preservice teachers in answering the questions. Unlike other studies (e.g. Widjaja, Stacey & Steinle, 2008)), participants did not have to choose from a list to explain their reasoning. Open responses were allowed therefore giving the possibility of getting a wider range of reasoning being applied in solving the problems.

Methodology

Data Sources

The data used for this study was collected from thirty-six (36) participants who were elementary teacher education students enrolled for the first part of a two-series mathematics content course at our university. Prior to enrolling in the class, every participant had taken at the minimum a College Algebra course.

A total of four questions were used for data collection purposes. Two of the questions were administered as part of a test that the participants took during the semester while the other two similar questions were embedded within the final exam for the course. The four question items, which are listed below, were specifically selected in order to detect preservice teachers' conceptions, errors and/or misconceptions about the questions of (a) identifying a decimal lying between two given decimals and (b) identifying a fraction lying between two fractions.

Questions

1) Name a decimal between 0.77777 and 0.77778. If it is not possible, explain why not.
2) Name a fraction between ¼ and ½ that has denominator 15.
3) Give a decimal between 4.768 and 4.769. If it is not possible, explain why not.
4) Name a fraction between $\frac{1}{7}$ and $\frac{2}{7}$. If it is not possible, explain why not.

Procedure

Questions 1 and 2 were included on a test while Questions 3 and 4 were among the questions on the final exam for the course. The researcher examined and coded participants' responses to each question, noting if each response was correct (1) or not (0). In addition, data on the procedure used to answer each question or explanation provided by each respondent were collected. The data were stored in a spreadsheet for analysis. Constant comparison method (Lincoln & Guba, 1985) was used to create categories of the explanations participants used justify their responses.

Results and Discussion

Table 1 shows how many of the thirty-six (36) participants correctly answered each question. Also shown in the table are the percentages of participants who correctly answered each problem.

Table 1

Percent Correct on om test

Question as it appeared om test	% correct ($n = 36$)
Question 1: Name a decimal between 0.77777 and 0.77778. If it is not possible, explain why not.	58.33% (21)
Question 2: Name a fraction between ¼ and ½ that has denominator 15.	94.33% (34)
Question 3: Give a decimal between 4.768 and 4.769. If it is not possible, explain why not.	63.89% (23)
Question 4: Name a fraction between $\frac{1}{7}$ and $\frac{2}{7}$. If it is not possible, explain why not.	69.44% (25)

Of the fifteen students who did not successfully address Question 1, only one did not provide a response while fourteen (14) said that it was not possible to have a fraction between 0.77777 and 0.77778. Figure 1 shows a response supplied by one of the participants, response that indicates the participant views 0.77777 and 0.77778 as being consecutive.

15) Name a decimal between 0.77777 and 0.77778. ***If it is not possible, explain why not.***

$\frac{7}{9}$ its not possible because there is nothing in between those 2 fractions. [2]

Figure 1. Erroneous argument on why there is no decimal between 0.77777 and 0.77778.

Another response in which the participant says there is no decimal between 0.77777 and 0.77778 is shown in Figure 2. Their argument seems to be that if you start with 0.77777, then the numbers to the immediate left and right of 0.77777 are 0.7776 and 0.77778 respectively.

15) Name a decimal between 0.77777 and 0.77778. ***If it is not possible, explain why not.***

There is no decimal between .77777 & .77778, if .77777 were changed it would become .77778, or .77776. [2]

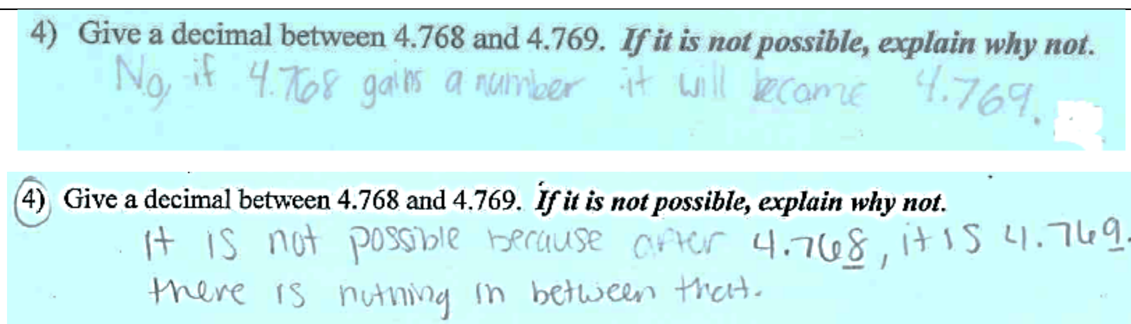
Figure 2. Faulty argument suggesting the real numbers 0.77777 and 0.77778 are consecutive.

Below are some additional explanations provided, verbatim in trying to justify that there is no decimal between 0.7777 and 0.77778.

Not possible because there is no number following 7 in the last place other than an 8. (2) There isn't any because the numbers are way too close together. (3) It's only the very last number that changes. (4) There is no decimal between because they come right after each other. (5) This is not possible because there no number between 7 and 8 meaning no number to place between the two numbers. (6) It is not possible because the two decimals are consecutive and hold same place value. (7) I don't think that is possible. 0.77777 has five places and 0.77778 is also. You can't put a decimal to make 0.77777.5 because it's already a decimal

The explanations provided in an attempt to justify why there is no decimal number between 0.77777 and 0.77778 seem to suggest that conceptions about natural numbers or integers were coming into play. Specifically, the participants seem to erroneously apply the fact that given two consecutive integers or natural numbers there is no other integer/natural number between them. The discrete nature of natural numbers often leads to a cognitive conflict with the compactness of rational numbers and the continuum of real numbers (Merenluoto, 2003).

Question 2 had the most success with about 94% of the participants giving the correct response. The two respondents who did not correctly answer this question provided fractions greater than ½. Of the thirteen students that did not successfully answer Question 3, seven said it was not possible to have a decimal between 4.768 and 4.769, four did not provide any response, one attempted and gave the wrong answer and one did not provide a numerical response but simply said it was possible. Figure 3 shows some of the responses in which participants argued that 4.768 and 4.769 are consecutive therefore there is no decimal between them.


4) Give a decimal between 4.768 and 4.769. ***If it is not possible, explain why not.***

No, if 4.768 gains a number it will become 4.769.

4) Give a decimal between 4.768 and 4.769. ***If it is not possible, explain why not.***

It is not possible because after 4.768, it is 4.769. there is nothing in between that.

Figure 3. Two incorrect arguments suggesting 4.678 and 4.769 are consecutive real numbers.

Eleven respondents failed to correctly answer Question 4. Three students provided incorrect numerical responses, four provided no response, while four reasoned that it was not possible to name a fraction between 1/7 and 2/7. Although not widespread in answering Question 4, Figure 4 shows work of a participant who converted 1/7 and 2/7 to 2/14 and 4/14 respectively. Although they wrote down the fraction 3/14, their conclusion was that it is not possible to name a fraction between 1/7 and 2/7 since the two fractions have the same denominator, which seems to suggest (as seen earlier) that they are looking at the numerators of the given fraction (which are consecutive) to reach a conclusion.

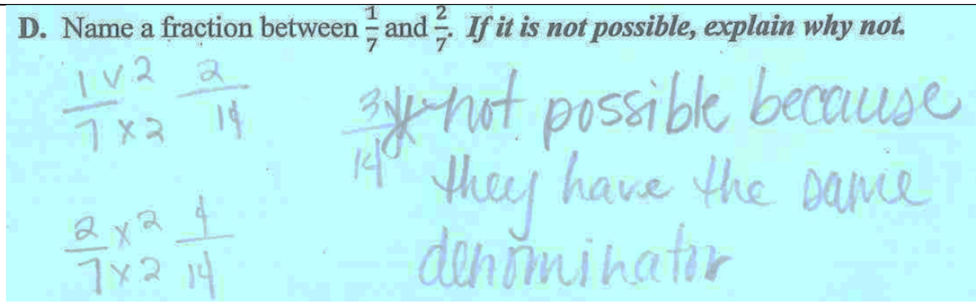


Figure 4. Faulty reasoning on why there is no fraction between 1/7 and 2/7.

The success rates in providing decimals between the decimals in Questions 1 and 3 are very close. When one looks at the percent of students who successfully provided a fraction between 1/4 and 1/2 and between 1/7 and 2/7 respectively, participants had less success in Question 4. The explanations provided suggested that because 1/7 and 2/7 have the same denominator and the numerators 1 and 2 are consecutive natural numbers, then there cannot be a fraction between 1/7 and 2/7.

We conclude this article by saying that the data analysis seems to suggest that some preservice teachers carry over conceptions about integers and natural numbers and erroneously apply them to problems involving rational numbers. What we can also infer from the results is that examples in the classroom and assessments on density of rational numbers should be varied in order to expose and let future teachers explore and reason with number concepts. Cramer and Lesh (1988) revealed that about a fifth of the participants did not understand rational number concepts well enough to meaningfully teach them. We echo the same sentiments as they did, that instruction should strive to make sure future teachers have the conceptual understanding of concepts they are expected to teach.

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INVESTIGATING PLACE VALUE CONCEPTS WITHIN A MEASUREMENT CONTEXT

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In the summer of 2017, sixteen students from an urban school district participated in a three-week summer course as part of a curriculum research and development project. The purpose of this project was to develop and implement lessons with incoming third-grade students focused on place value concepts presented within different number systems developed through generalized quantitative contexts (using mass, area, length, and volume). Lessons were designed, implemented, and retrospectively analyzed using design research methods to examine and further develop the curriculum. This qualitative study reports initial findings about the mathematics with which the students engaged that informed development of the curriculum.

Theoretical Framework and Related Literature

Place value concepts play a fundamental role in students’ understanding of number sense, operations, algebraic reasoning, and beyond. Typically, concepts in place value are rooted in ideas of counting and cardinality in the decimal number system. Consequently, prior studies on the development of number skills have focused on students working solely in base ten and using discrete models. In addition to number naming strategies, other indicators of place-value understanding have focused on the application of strategic counting skills (i.e., Chan & Tang, 2014; Ho & Cheng, 1997) and the ability to compare magnitude differences between number pairs (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011). In contrast, this study is based on the premise that experiences with continuous quantities serve as a natural starting point for mathematics learning because it mediates learning of concepts that are primary and basic in the structure of mathematics and students’ ways of making sense of the world (Davydov, 1975a, 1975b, as cited in Venenciano & Dougherty, 2014). Research into student thinking within such an approach suggests that they develop insight into place value concepts and the magnitude of numbers through a generalized measurement context. In Slovin and Dougherty's (2004) study of second grade students who were learning to represent counting numbers in different number systems, they found that while some students were operating procedurally and specifically, there was evidence that others were operating conceptually in their reasoning about how a place value system worked. In another study, written responses from 30 students on two problems from an assessment given from 2002–2008 highlight how students showed their understanding of place-value from a generalized quantitative approach. Responses to the first problem showed their ability to create supplemental units within a base, and responses to the second problem revealed students' insights into number systems (Venenciano, Slovin, & Zenigami, 2015). These studies highlight how working with place value concepts within a generalized quantitative context focuses student attention on the structure of number systems, a foundational construct of place value.

Our study builds upon this earlier work and examines how students with basic experiences working with continuous quantities (e.g., mass, volume, length, area) grapple with measurement tasks designed to promote their understanding of place value concepts. In particular, we focus on this question: How can a quantitative approach involving continuous quantities support students' learning of place value concepts? We hypothesized that working with different number systems in quantitative contexts provides students with opportunities to make generalizations about the structure of number systems and to develop an understanding of place value through a non-discrete context.

Development of the Curriculum

Thirteen lessons were developed based on an approach put forth by Davydov (1975a, 1975b) as a way for children to develop a robust understanding of place value. Using design research methods (Gravemeijer, 1994), the developed lessons were analyzed as they were enacted within the Hawai'i Elementary Mathematics Laboratory (HEML), a three-week summer class for students entering third grade. HEML afforded researchers and curriculum developers the opportunity to implement learning experiences immediately following lesson design sessions. The implementation of the lessons provided opportunities to analyze their enactment in the classroom and to continue the curriculum development based on students' mathematical thinking and experiences during the lessons. The curriculum development began with outlining a "possible learning route that aims at significant mathematical ideas and a specific means that might be used to support and organize learning along this route" – the beginnings of what Clements refers to as a "hypothetical learning trajectory." (Clements, 2008, p. 595). The foci of lessons 1–6 included introducing the students to the class context and building the culture of the classroom community. Students were also introduced to working and measuring with the different continuous quantities (mass, volume, area, and length). In lessons 7–13, place value concepts were the main foci; these are highlighted in Table 1.

Table 1

2017 Project Hypothetical Learning Trajectory Concepts

	Mathematical Foci
Lesson 7	• Exchanging units: A predetermined number of smaller units can equal a larger unit of a continuous quantity (area, length, volume or mass).
Lesson 8	• Introduction to multi-digit numbers and place value in base five. • Given a main measure, supplemental measures can be made using volume for this base. These measures can be used to make a quantity.
Lesson 9	• Introduction to multi-digit numbers and place value in base three. • Given a main measure, supplemental measures can be made using volume for this base. These measures can be used to make a quantity.
Lesson 10	• Given a main measure, supplemental measures can be made for any base. These measures can be used to make a quantity.
Lesson 11	• Other base number systems: A base number system can be determined by finding the relationship between the main and supplemental measures. • Counting or measuring in smaller base number systems establishes a structure for subsequent work in base ten.
Lesson 12	• Introduction to base ten: Supplemental measures for base ten are introduced.
Lesson 13	• Continued work in base ten

Immediately after the enactment of each lesson, the teacher and observers met to discuss and review students' work from the lesson. The meetings focused on retrospectively analyzing students' mathematical thinking and experiences as well as on refining subsequent lessons. Retrospective analysis involves situating the design experiment within a "broader theoretical context" that results in "accounts of learning that relate learning to the means by which it can be supported and organized" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 13). In the curriculum development team's discussions, students' thinking and experiences were discussed within the framework of Davydov's approach. The next lesson was then adjusted and refined.

Methodology

The three-week summer program took place in June 2017 with a diverse group of 16 students from four public elementary schools located in an urban area with a high-needs population.[i] The research team consisted of a teacher-researcher with 17 years of elementary school teaching experience but new to teaching from an approach using continuous quantities, two curriculum researchers who were involved in developing the original lessons from which the summer materials were adapted, and a researcher with expertise in practice-based research involving multimedia records of practice. The team worked with principals and curriculum coordinators from the schools to identify students for the program. In order to encourage participation, tuition was waived and school supplies for the class were provided. The district in which these schools reside utilizes a state-mandated curriculum aligned to the Common Core State Standards that is based on a counting-first approach to learning place value concepts within the decimal number system.

The research team collected records of practice from each lesson comprising a video recording of the lesson, still photos, digital scans of the written work produced by students during the lesson, and the lesson plan and materials prepared by the teacher. Researchers also conducted pre- and post-interviews and a drawing task with the students. The team reviewed the video, identifying the activity segments that comprised each lesson. Segments focused on students' thinking about place value were selected, transcribed, and analyzed for evidence of their evolving understanding of place value concepts, their ability to work with and use different number systems, as well as their capacity for reasoning, explanation, and justification. This qualitative study reports on the results of this analysis of lesson video recordings – particularly of students' experiences with place value concepts in whole class discussions where they explained their solutions and thinking about place value.

Findings

Making Exchanges (Lessons 6–8)

A foundational concept to place value and arithmetic introduced in Lessons 6 and 7 involves the notion that iteration of a unit can make a larger unit. In Lesson 8, students used a continuous quantity of area, iterating a small unit to create an area which could then be represented as a new larger unit to reinforce this idea.

Area Table

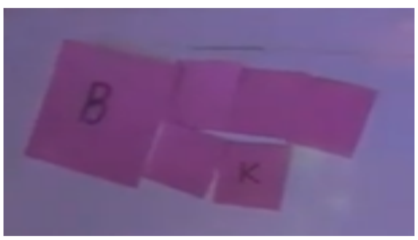
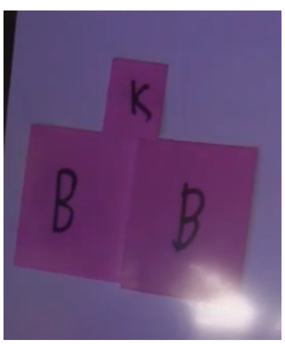
$B \xleftarrow{4} K$		
1	5	*L*

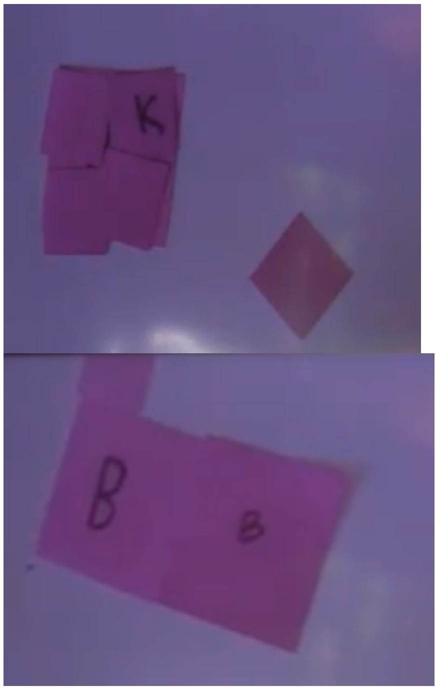
Fig 1: Learning task about Exchanging Units

The table in Fig. 1 shows area-unit *K* is used four times to make area-unit *B*. Area *L* is made with five area-units *K* and one area-unit *B*. Students were provided with four area-units *B* (2 in. x 2 in. squares) and six area-units *K* (1 in. x 1 in. squares) to construct an area *L*. Table 2 includes examples of area *L* made by two different students. Student 1 created area *L* with five area-unit Ks and one area-unit B, which is accurate by the information provided, but did not account for the notation indicating four area-units *K* make an area-unit *B*. Student 2 created an area in which four area-units *K* were exchanged for one area-unit *B*. In his explanation, he placed four area-units *K* on an area-unit *B*, to support his understanding of the equivalent relationship ($4K = B$), but he described the action as taking away and adding back. The teacher provided the language of "exchange" to more accurately describe the mathematics of his action. It was during this interaction that the concept of exchanging units became explicit for the class.

Table 2

Student Responses to Exchanging Units Task (Lesson 8)

Example	Transcript
	Student 1: So our area *L* is four pieces, combined with one more. Which is five *K*s, and this is how area *L* looks like.
	Student 2: I put two *B*s because, um, I had five *K*s and to make a *B* you need four, so, I used the four *K*s to make one *B* and there was one left so I put it as a *K*. And that's it. T: It might help if you show us how you started. So, here's the area unit *K*s. Can you show us, how you started? St. 2: There's one and the other four is in here (points to area *B*) so got five, I took apart from four, and um, I made another *B*, and there's one left, I put back the *K*. T: So, you're saying you took four of the *K*s and put it together into a *B*. And there was one more, and that's that extra *K*. St. 2: There was five *K*s (places four area-units *K* on area *B*), but I took away four and added T: Okay, wait, where's the five area-units *K*? St. 2: These four and this one. And then I took this thing away



(takes away the four area-units *K*) then I added another *B* then I put this (places one *K* in the area)

T: So you did an exchange.

St. 2: Yes.

Supplemental Measures (Lessons 8-11)

Students were introduced to place value through smaller base number systems to help them understand the structure of place value of multi-digit numbers throughout Lessons 8–11. Base five was introduced through a story context involving aliens from an imaginary planet, Quinar, who only had the symbols 0, 1, 2, 3, and 4. Students pretended to be from Quinar and were asked to determine length *A* (19 length-units *E*). While counting length-units, they realized there is a problem at length 5*E* since no symbol “5” exists on Quinar; the supplemental unit E_{II} was introduced to replace 5*E* which set the foundation for multi-digit representations. Working with smaller bases helps students see the need to exchange to the next place value and generalize place value concepts through measuring large quantities. They see that, depending on the base system, they must create a supplemental unit in order to numerically represent the quantity. For Lessons 8–11, students were introduced to different number systems and various measuring tasks which required them to work within those number systems by creating supplemental units and either measuring or creating a given continuous quantity such as an area or length.

As an example, students were tasked to use supplemental units in a given number system to measure a quantity of area. One pair of students working in base five combined their main unit and supplemental unit, iterating them together four times, to measure the area, resulting in four main units and four supplemental units, or spoken “four-four base five” (44_{five}). Another student used 24 base six main units to measure the area and arrived at “twenty-four” in base six, and overlooked the fact that counting in base six would produce a different result than counting in base ten. Introducing and working with supplemental measures in different number systems provided a conceptual foundation for working with place value in base ten.

Supplemental Measures in Base Ten (Lessons 12–13)

During the last two days of instruction, attention turned to base ten. The tasks in Lesson 12 involved creating supplemental units in base ten. Students worked with area to make an E_{II} supplemental unit in base ten, then predicted how an E_{III} unit might be created in base ten. While most students were successful making E_{II}, several thought $3E_{II}$ would make E_{III}. However, one student connected her prior experience with other bases and described, "We used 10 of the *E*s to make E_{II}, and we usually use the same number of E_{II} to make E_{III}," to which most of the class then agreed.

On the final day, students reviewed the base ten supplemental units, including making a class supplemental E_{IV} base ten area unit. One student observed, “So whatever base it’s on, that’s how much of the *E*s you’re gonna need to make the E_{II}. So if it was on base three, you would use three bases to make an E_{II} and you would use three bases to make E_{III}.” While her vocabulary may not yet be accurate, she appears to be thinking in a generalized way about the relationship of places in multi-digit numbers of any base.

Discussion and Implications

The students we worked with had no prior experience with the lessons and tasks from this curriculum, and, as we expected, were relatively new to measuring with continuous quantities as a means for learning place value concepts. As such, students encountered a number of challenges over the course, which had curricular and instructional implications for the lesson designs. One challenge involved students’ unfamiliarity with length, area, mass, and volume. A few students struggled with recognizing that they could measure an amount by iterating a unit, and they did not always see the importance of precision in measuring. In addition, working in different number systems within a brief amount of time was challenging – as well as making the quick transition to base ten. It is possible that more explicit instruction focused on the digits that are used within a base and on mathematical vocabulary is needed to support student learning.

Working with place value through measuring continuous quantities within different number systems places the mathematical focus on generalized concepts, a notion foundational to the approach put forth by Davydov (1975a, 1975b) and his colleagues. Over the three-week period with this class, students began to attend to the structure inherent to number systems, with two children making generalizations based on their experiences with measuring continuous quantities. The results from this project provide us with information for furthering our curriculum development efforts based on this approach for learning mathematics.

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[i] In the 2016-2017 school year, eight elementary schools (K-5) in the state had a lower percentage of "math proficient" students than the state average. Five of these schools were in this urban area.

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PROPORTIONAL REASONING: STUDENT THINKING AND IMPLICATIONS FOR TEACHING

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Our research in the acquisition of proportional reasoning showed that task difficulty and strategy use is predictable, given the classification of a task's number structure. In this work, we translate our research to practice, that is, we describe how teachers can use our findings. We first describe mathematical knowledge for teaching as it pertains to the topic of ratio and proportion and then categorize number structure characteristics that affect student thinking. Teachers can use our categorization and findings to enact the teaching practice, "elicit and use evidence of student thinking" (NCTM, 2014, p. 10) to assess progress and adjust instruction.

Beyond investigating fascinating questions into how people think, we want our mathematics education research to positively impact learners. The mathematical knowledge for teaching model (Ball, Thames, & Phelps, 2008) indicates that teachers need knowledge concerning the *mathematics* of a topic, the *tasks* that engage students with the mathematics, and typical *student responses* (their strategies and their thinking). We examine these three types of knowledge to translate our research in the acquisition of proportional reasoning in students to knowledge that informs practice for teachers. More specifically, we ask, "What do teachers need to know to use missing value proportion problems to develop proportional reasoning in their students?"

Related Literature

We begin by reviewing background material on the mathematics of proportion, the missing value task, and the variety of student-generated strategies.

Proportional reasoning is addressed in middle school, but its foundation is laid much earlier. Students grapple with proportionality when making sense of the base 10 number system and they use multiplicative comparisons in rational number contexts. A proportion is the equality of two ratios, denoted mathematically as $A/B = C/D$. *Essential Understandings of Ratio and Proportion* (Lobato & Ellis, 2010) identifies the key aspects of the underlying mathematics. The anchoring sentence is this: "When two quantities are related proportionally, the ratio of one quantity to the other is invariant as the numerical values of both quantities change by the same factor" (p. 11). The mathematical goal is to make sense of two types of ratios and the multiplicative structure of the proportion. We will unpack this goal further when we discuss evidence of understanding wthat student work provides.

To make sense of the two types of ratio, we must first distinguish them. Contextual problems, rather than purely numeric ones, make this task straightforward. To name the ratios, consider the example in which *A* boxes of treats are needed for every *B* people. The ratio *A: B* is *between* two types of quantities, boxes and people, and so is called a *between measure spaces ratio*. The other ratio is a ratio of two quantities of the same type, as when comparing the original number of boxes (or people) to a scaled number of boxes (or people). This ratio is called the *within measure space ratio*. *Scale factor* is a common term for the numerical value of this ratio. The between measure spaces ratio, numerically, is the constant of proportionality or the invariant of the proportion. This number is the functional relationship between the quantities (that is, the k in $y = kx$) and appears as the slope in a graph of the relationship. *Unit rate* is a related idea. Other terms exist for the between and within measure space ratios; we prefer these terms as they can be distinguished using the units of the quantities. This vocabulary is for researchers and teachers, not students. Students articulate the ratio by referring to the kind of quantities they compare.

Our work uses missing value proportion problems to engage students in proportional reasoning. This type of problem is one in which three quantities are given and the goal is to find the fourth so that a proportion is formed. An example is: *If 4 boxes of treats are needed for every 12 people, how many boxes are needed for 42 people?* This type of task is ubiquitous in textbooks, in every day mathematics, and in mathematics research. These tasks are structurally simple, hence easy to create. Researchers, however, know that many of their features, such as context, semantic type, and choice of numbers, affect student responses. Number features are very influential. Teachers need a practical way to identify suitable tasks for their students.

We designed our research to study student success rates and solution strategies on missing value tasks with various number features. See (Riehl & Steinthorsdottir, 2017) for investigation methods and findings. We classify problems by characterizing each ratio as an integer or non-integer ratio. In the example task, the between measure spaces ratio is 4: 12. This is an **integer** (I) ratio since the larger number is a whole number multiple of the smaller. The within measure space ratio is 12: 42, which is a **non-integer** (N) ratio. The target situation (X boxes: 42 people) is an enlargement of the original, so the task is an “IN enlarge” problem. In denoting the classification, the between measure spaces ratio is always given first. An “IN shrink” task is: *If 14 boxes of treats are needed for every 42 people, how many boxes are needed for 12 people?*

Table 1 displays the eight possible number structures using the characterizations of the two ratios and the direction of change, along with an example proportion for each classification cell.

Table 1

Number structure characteristics

		Within measure space ratio	
		Integer	Non-integer
Between measure spaces ratio	Integer	Cell A: II Enlarge or shrink $\frac{4}{12} = \frac{20}{60}$	Cell C: IN Enlarge or shrink $\frac{4}{12} = \frac{14}{42}$
	Non-integer	Cell B: NI Enlarge or shrink $\frac{4}{10} = \frac{20}{50}$	Cell D: NN Enlarge or shrink $\frac{4}{10} = \frac{14}{35}$

Students learn mathematics by grappling with appropriate tasks and they reveal their understanding in the strategies they use. Teachers are encouraged to "elicit and use evidence of student thinking" as one of the eight Mathematics Teaching Practices (NCTM, 2014, p. 10). Cognitively guided instruction developed this approach to assess students' progress in learning basic operations (Carpenter, Fennema, Franke, Levi, & Empson, 2015). Students generate strategies based on what they know. Teachers guide students to correct erroneous reasoning and develop conceptual understanding before encouraging abstraction and use of algorithms. Similarly, teachers can assess and guide students' progress in proportional reasoning by evaluating their work on missing value proportion tasks. Teachers need evidence that students understand the multiplicative structure of proportion (and so can use both ratios in solution strategies) and are able to articulate their reasoning and apply it in a variety of settings. By reasoning through thoughtfully chosen problems, students deepen their understanding of rational number as they explore transformations that maintain the invariant of the proportion. This study of a mathematical structure will be extremely beneficial as students continue to algebra and higher mathematics. The too-early introduction of an algorithm may undermine the learning process and lead to overgeneralization (Van Dooren, De Bock, Evers, & Verschaffel, 2009) and lack of understanding.

Student strategies may be erroneous or valid. Erroneous strategies include haphazard or incomplete calculations and absolute rather than relative comparisons. Valid student-generated strategies for missing value proportion problems include build-up and multiplicative strategies. Correct strategies may be rudimentary and inefficient or be sophisticated and efficient. See Riehl & Steinthorsdottir (2017) for examples. Partial understanding of a proportion, unfortunately, can give the impression of mastery. Students may comprehend only one of the two ratios, yet have a valid strategy for all problems. Teachers must look for evidence of understanding of both ratios to ensure students gain the desired depth of knowledge. In the next section, we unpack the mathematical goal and identify elements of student work that provide evidence of understanding. In the final section, we demonstrate how teachers can develop and extend students' understanding of proportion by making intentional number choices in the missing value tasks they pose.

Student Work as Evidence of Understanding

Delineating components of the knowledge needed to understand ratio and proportion is likely impossible. Yet a continuum of understanding does exist and in order to assess progress, we need to identify significant points. A satisfactory outcome of understanding is recognizable: Students are able to identify when quantities are related proportionally and are able to flexibly choose whether to use the multiplicative relationship within or between the measure spaces to solve proportion related tasks.

An initial step to reason proportionally is to form a composed unit. The student knows that the given quantities are linked and must be treated in some parallel fashion. Further, the student knows to use relative comparisons rather than absolute comparisons. Students with this level of understanding can implement a valid strategy. If students are not linking the quantities, they may perform haphazard calculation (adding the three quantities, say) or compare two quantities and ignore the third. If students use absolute comparisons, then they are trying to equate A: B and (A + c): (B + c); this is sometimes called the additive error strategy. Consider the task: *If 4 boxes of treats are needed for every 12 people, how many boxes are needed for 42 people?* The additive error strategy yields the answer 34.

Next, the student employs one of the two multiplicative relationships. The use of the within or between measure spaces ratio can take many forms. The numbers in the task, specifically whether the ratios are integer or non-integer, influence the students in their choice of strategy.

Students tend to be more successful when integer ratios are available. Table 2 compiles indicators and examples to the questions teachers ask to assess student understanding. Student work will take many forms and a solution is likely to include multiple indicators.

Table 2

Indicators of Understanding

Query	Indicators	Example using 4 boxes: 12 people
Does the student recognize the quantities are related proportionally?	The student forms a composed unit. The student uses relative comparisons.	The same operation performed on one quantity is performed on the other.
Does the student understand the multiplicative relationship of two quantities *within* the same measure space?	The student creates equivalent ratios by iterating the composed unit. The student creates equivalent ratios by partitioning the composed unit. The student creates equivalent ratios by scaling the composed unit.	$\frac{4}{12}=\frac{8}{24}=\frac{12}{36}=\cdots$ $\frac{4}{12}=\frac{2}{6}=\frac{1}{3}=\cdots$ $\frac{4}{12}=\frac{4\ \cdot c}{12\ \cdot c}$
Does the student understand the multiplicative relationship *between* the quantities in different measure spaces?	The student creates a unit rate. The student creates equivalent ratios using the invariant.	1 box per 3 people 1/3 box per 1 person $\frac{4}{12}=\frac{b}{3\cdot b}=\frac{\frac{1}{3}b}{b}$

Implications for Teaching

Our project investigated both the success rate and strategy choices of over 400 middle school students on missing value proportion tasks in which we intentionally varied the number structure.

We found both the difficulty of the task and the types of strategies chosen by students are predictable, based the classification given in Table 1 (Riehl & Steinthorsdottir, 2017). In general, shrink problems are more difficult than comparable enlarge problems and the difficulty of tasks increases in order from Cell A (II) and B (NI) to Cells C (IN) and D (NN).

An obvious implication for teaching is that the numbers in the task matter. A more helpful implication: Our framework of classifying missing value proportion problems (Table 1) allows teachers to organize their intuitive knowledge about the difficulty of missing value problems and relate difficulty to specific problem characteristics. A given task is simple to classify and for a problem stem, teachers can create additional tasks of predictable difficulty with minor adjustments to the numbers. Students need many experiences with proportional reasoning tasks and encountering the eight number structures ensures important variety. By making intentional choices of the numbers, teachers can use missing value proportion tasks to develop students' understanding of the two multiplicative ratios in a proportion.

Another finding of our research is the change in difficulty from cell to cell is not uniform. In particular, Cell C (IN) tasks are significantly more difficult than tasks from Cell B (NI) despite both cells containing tasks with exactly one integer ratio (Riehl & Steinthorsdottir, 2017). This suggests that students do not develop understanding of the two types of ratio in the same manner or at the same time.

A rudimentary strategy is to iterate the given ratio and so to build up to the target. This focuses students' attention on the within measure space ratio as they are comparing quantities sharing the same units. Because they have formed a composed unit, operations performed on one type of quantity are duplicated for the other type. This strategy works well when the scale factor is an integer (Cells A and B). In this case, students do not need to notice the relationship between quantities of different types. Eventually, students use multiplication rather than repeated addition and efficiently compute the target value. When the scale factor is a non-integer (Cells C and D), a student does not necessarily realize that the scale factor can still be computed and applied. Instead, students may scale by a whole number to get close to the target, and then figure out how to handle the "leftovers."

Knowledge that allows the students to partition (scale down) the unit is distinct from the knowledge that allows them to iterate it. This additional knowledge becomes a tool that allows students to solve shrink problems. It also provides a route to handle leftovers in build-up strategies. Partitioning the unit is especially convenient when there is an integer relationship between the quantities (Cells A and C). In this case, a unit rate is evident. Students might not realize that the relationship between these two quantitates provides another route to the solution. Instead, students might scale the two quantities in the new ratio. Our research provides evidence that students often fail to notice and use the between measure spaces ratio and instead prefer to scale quantities, even when the numbers are inconvenient. The appearance of a unit rate will facilitate conversations regarding the relationship between the two types of quantities. This is an important step on the route to understanding proportions and teachers should create opportunities for students to explore the invariance of this factor.

Here are brief examples illustrating that through intentional number choices, teachers can support and extend their students proportional reasoning. The context of the tasks is omitted, but should be created when given to students.

- With beginning students, a learning goal might be to explore multiple ways to solve a missing value task. A Cell A task, such as 6: 18 = 30: X, is likely to generate a variety of strategies. Teachers will have the opportunity for rich discussions comparing and contrasting the strategies.
- Suppose many students can successfully solve tasks by iterating the composed unit. The teacher has the learning goal for them to apply their knowledge of multiplication as repeated addition to make the build-up method more efficient. Try a task from either Cell A or Cell B with a large scale factor, such as 5: 4 = 200: X. This will encourage students to look for multiplicative relationships.
- Suppose the teacher has the learning goal for students to partition the composed unit. A shrink task from Cell B, such as 42:14 = 6: X, or from Cell A, such as 24: 6 = X: 2, is appropriate.
- If the learning goal is for students to notice and use the between measure space ratio, a Cell C task with messy scale factor and an appealing unit rate, such as 6: 18 = 22: X, will elicit multiple solution strategies. The teacher will be able to facilitate conversations drawing students' attention to both multiplicative relationships.

One of the goals of research in mathematics education is to improve student learning. To reach that goal, researchers must translate their academic findings to practice. Student learning improves when well-prepared teachers are in the classroom, and well-prepared teachers have much mathematical knowledge for teaching. Increasing mathematical knowledge for teaching begins a virtuous cycle: When teachers dig into the mathematics of proportion, they can specify learning goals for their students. As they dig into the tasks that engage their students in proportional reasoning, teachers improve their understanding of how changes in a task's features affect student thinking. As they gain experience in interpreting student work, their ability to assess students' progress increases. With the knowledge of what their students understand and what they still need to learn, teachers can create sharper learning goals and choose appropriate tasks. The cycle continues.

Our research examined how student success rate and strategy use varied as the number structure (Table 1) of missing value proportion tasks changed. Our aim now is to help teachers use the number structure of proportion as they pose tasks and interpret student thinking.

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AN INQUIRY-BASED APPROACH TO MATHEMATICS

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Despite the significant amount of research on inquiry-based methods (e.g., problem-based, project-based learning), there is only a scarce amount of research on them in secondary education. The purpose of this study was to examine the perceptions of secondary school mathematics teachers on their facilitator training, classroom experiences, roles, skills, and implementation challenges on problem-based learning. Survey data revealed that most participants indicated that their training was effective at helping them understand the philosophy of the teaching and learning approach, and the training provided them with sufficient insight into how to manage the small group learning process.

Introduction

Instructional practices over the years in mathematics classrooms have not varied significantly. In addition, they have not adequately aided students in developing a deep understanding of mathematical ideas. Mathematics education stakeholders continue to seek a methodology that will best educate learners of the 21st century since many students in the mathematics classroom are lacking in their ability to understand, communicate, and apply key concepts in mathematics (O'Brien, 1999; 2004). In an attempt to meet the needs of today's diverse learners, some national organizations are urging classroom teachers to use innovative methods of instruction that aid students in demonstrating comprehensive learning and apply it to real world settings (National Council of Teachers of Mathematics, 2000; National Science Foundation, 2006).

Education stakeholders are now placing an emphasis on students' ability to understand and use information, not just merely possess it (Richardson, 2003). According to many researchers and practitioners, problem-based learning (PBL) is an innovative inquiry-based, viable instructional approach for teaching mathematics that can aide students in reaching these significant learning goals (Erickson, 1999; Lubienski, 1999; Ronis, 2008). Consequently, the purpose of this study was to examine the perceptions of mathematics teachers of their PBL professional development and implementation of PBL into their classroom.

Theoretical Framework and Literature Review

Problem-based learning has its roots in constructivism, but many of these views date back to John Dewey (1938). While PBL has a foundational framework in medical education, it is consistent with the principles of constructivism (Torp & Sage, 2002). In classrooms using problem-based learning, teachers address these principles of constructivism, and Savery and Duffy (1995) have argued that PBL learning environments may be one of the best examples of a constructivist learning environment.

Research in PBL usually focuses on whether students who are taught with PBL learn as much as students who are taught with a traditional instructional approach (Gallagher & Gallagher, 2013). Research supports that students in PBL classrooms can learn as much as or more than students taught using a traditional method if the problems are closely aligned with content objectives (Gallagher & Stepien, 1996; Goodnough & Cashion, 2003). Students must also be provided with appropriate support (Hmelo-Silver, Duncan, & Chin, 2007; Vardis & Ciccarelli, 2008) for PBL to be effective. Research specifically related to secondary school mathematics students or secondary mathematics teachers is scarce.

However, a recent study in secondary school mathematics regarding students' academic skill development and motivated strategies for learning supports that PBL has a positive impact on students' learning. Results showed that at-risk and minority students benefited significantly from PBL in learning mathematics. Though the academic performance gap was present, it was significantly reduced. Also, PBL students were more intrinsically motivated and showed significantly higher critical thinking than their public-school counterparts (Holmes & Hwang, 2016). Though there are a few other studies related to PBL in the mathematics classroom (Cerezo, 2004; Clarke, Breed, Fraser, 2004) more investigations are needed.

Methodology

An online survey approach was selected to collect the data since many participants were geographically disbursed. Another advantage of the online survey was that it insured anonymity and confidentiality. Moreover, it allowed respondents to complete the survey online when it was convenient for them increasing the chance of a high response rate.

Participants

The target population in this study was secondary mathematics teachers who had completed a PBL professional development workshop. The sample participants were purposefully selected based on these criteria. This sample consisted of teachers who were members of the Problem-based Learning Network (PBLN) at the Illinois Mathematics and Science Academy (IMSA). PBLN provides professional development to PBL facilitators, who range in experience from novice to expert, on inquiry-based teaching strategies and skills. Teaching and facilitation experience are indicated in Figure 1.

The PBL training ranged from a one-day session to a two-week summer session. The facilitators participated as learners (or students) engaged in a problem scenario. Therefore, they had the opportunity to experience PBL in a way that is similar to how their students would experience it. They also engaged in the stages of designing a PBL scenario for their classrooms and collaboratively developed strategies to implement them effectively into their classrooms. They gained first-hand experience in facilitating or coaching strategies while identifying learning objectives linked to state and national standards and benchmarks. There were 75 secondary mathematics teachers who had gone through the PBLN training. All PBLN participants who responded to the survey were included in this study.

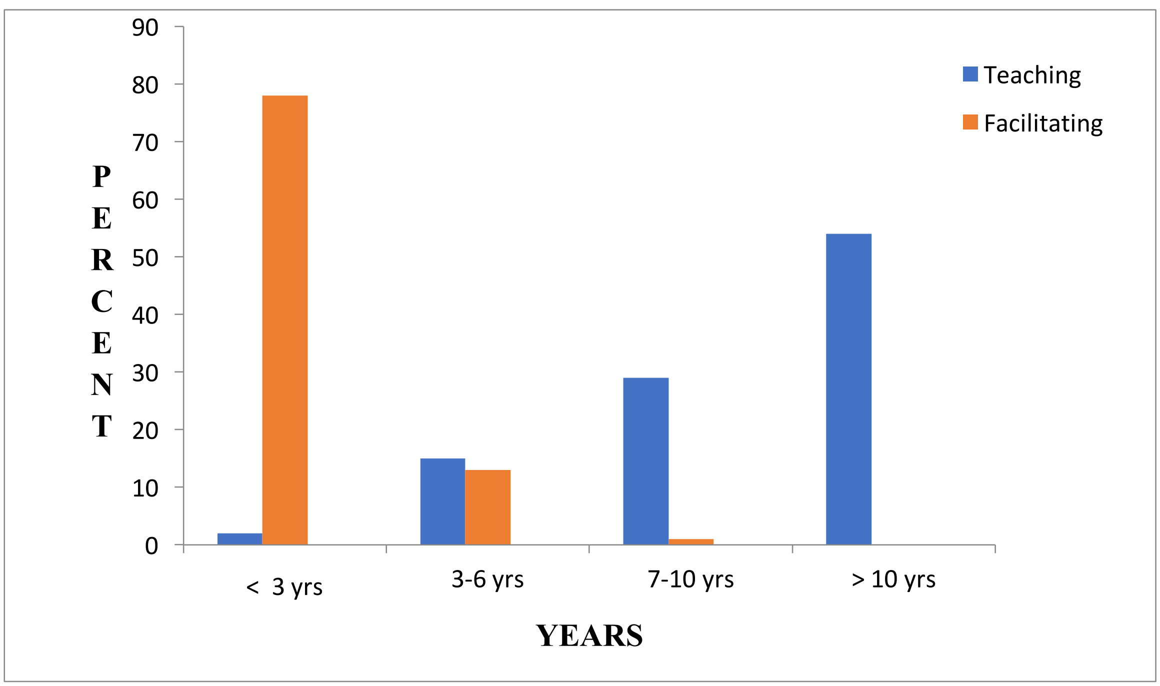

Figure 1. Teaching and Facilitation Experience of PBL Teachers.

Instrument

Likert scale questions for the *Facilitator Perception Survey-Revised* were adapted from a survey used in a study by McLean (2003) in which he categorized items under four key facilitation competencies (i.e. facilitation skills, curriculum knowledge, personal qualities, and subject-matter expertise). For the present study, revisions were made to the original instrument based on a review of the literature on PBL in secondary education, and validity and reliability results were reported. The revised survey includes questions that assessed participants' demographic characteristics as well as questions that related to participants' perceptions of the facilitator's roles, skills, training, classroom experiences, and implementation challenges. For many questions, participants were asked to respond using a continuum scale where 1 = effective, 2 = weak, 3 = satisfactory, 4 = good, and 5 = excellent.

There are also six open-ended questions on the revised survey that assessed the facilitator's perception of PBL. Sample questions include (a) what motivated (or prompted) you to implement PBL in your classroom (e.g., believed in the PBL philosophy, my head of Department/School asked me to implement PBL); (b) based upon your facilitation experience, describe the strengths and weaknesses of the PBL approach to teaching; (c) describe any challenges and/or frustrations you have experienced in the PBL facilitation process.

Procedure

The sample consisted of 75 secondary school mathematics PBL facilitators, which revealed a 55% (*n* = 41) response rate. All participants completed the standard demographic information (i.e. gender, age, in what state do you live, highest level of education, number of years teaching mathematic). The data were analyzed using SPSS, version 21. As illustrated in Table 1, a demographic profile emerged from the survey sample indicating a heterogeneous sample in a number of categories (e.g., age, gender, education, teaching experience, and facilitator training).

Table 1

Survey Participants Demographics

Survey Item	Response	n	Percent
Teaching Experience	>10 years	22	54
	7-10 years	12	29
	3-6 years	6	15
	< 3 years	1	2
Facilitator Experience	>10 years	0	0
	7-10 years	1	3
	3-6 years	5	12
	<3 years	32	78
	Missing	3	7
Number of Problem Facilitated	< 2	19	48

	2-5	16	40
	>5	5	12
	Missing	1	3
Highest Level of Education	Bachelor's Degree	6	15
	Master's Degree	34	83
	Education Specialist Degree	1	2
Age	<40	13	32
	41-50	22	54
	>50	6	14
Gender	Female	27	66
	Male	14	34

Results and Discussion

Research Question 1

Three questions on the survey focused on participants' perceptions of their PBL training. One question asked participants to rate their knowledge or understanding of the PBL philosophy, prior to PBL training. The responses to this question indicated that before PBL training, 7.5% (n = 3) of the respondents rated their knowledge or understanding of the PBL philosophy as good or excellent. On the other hand, when asked to rate their knowledge or understanding of the PBL philosophy after PBL training, 80.7% (n = 32) of the respondents rated it as good or excellent. Forty-four percent (n = 17) agreed or strongly agreed with the statement: "I feel/felt confident before facilitating my first session," but an equal percentage (44%) of participants (n = 17) were not sure or disagreed with this statement. Also, two participants chose not to respond to this statement.

Less than half (47.5%) of the participants (n = 19) agreed that their first experience facilitating was a success, and an equal percentage (47.5%) of participants (n = 19) felt confident after their first facilitation experience. Once the participants facilitated one theme, 65% (n = 26) agreed or strongly agreed that the PBL training made more sense in terms of understanding the role of the facilitator in small group learning sessions, and only 5% disagreed (n = 2).

Research Question 2

Research question two addressed whether there were any differences in the perceptions of roles and responsibilities among secondary mathematics PBL facilitators with different levels of facilitation experience (i.e., novice, intermediate, advanced, and expert). To determine if there were mean differences between the novice and advanced, secondary mathematics PBL facilitators on their *roles* scores, the ANOVA test was conducted. The mean *roles* score in the novice group ($M = 3.33$, $SD = .44$) was slightly lower than the mean *roles* score in the advanced group ($M = 3.39$, $SD = .08$). The results of the ANOVA test revealed $F(1, 38) = .22$, $p = .64$. Although the mean score for the advanced group was larger than the mean score for teachers in the novice group, the results indicated that there were no differences in the perceptions of roles and responsibilities among secondary mathematics PBL facilitators with different levels of facilitation experience (i.e., novice and advanced). At an alpha of .05, this finding supports that there is no statistically significant difference between mean *roles* perception scores of novice and advanced secondary mathematics PBL facilitators. The effect size statistic, eta-squared, revealed .0056, which indicates a very small effect based on Cohen's (1988) guidelines. However, this value indicates that .56% (less than 1%) of the variance in the *roles* score can be explained by facilitation experience (number of problem scenarios facilitated).

Research Question 3

Research question three addressed whether there were any differences in the perceptions of the skills needed for effective implementation of PBL among secondary mathematics PBL facilitators with different levels of facilitation experience (i.e., novice and advanced). Like the previous research question, analysis of variance (ANOVA) was used to answer this research question. At an alpha of .05, this finding supports that there is no statistically significant difference between the mean *skills* perception scores of novice and advanced secondary mathematics PBL facilitators. The effect size statistic, eta-squared, revealed .0411, which indicates a small effect based on Cohen's (1988) guidelines. Given the small effect detected, it is possible that the sample size in this study might have influenced the results.

Research Question 4

Research question four addressed the challenges secondary school mathematics, PBL facilitators perceive they encounter while they are planning for and implementing PBL into the classroom. Twenty-five participants responded to the invitation to comment on this open-ended question: Describe any challenges and/or frustrations you have experienced in the PBL facilitation process. The most frequent comments were related to challenges concerning the roles of a facilitator. Thirteen participants commented on the difficulty and challenge of their roles. For instance, this is how one participant described the issue: "The students have had little experience of working together in a group. The process of trying to teach them how to work together while doing the problem was very difficult." At the same time, another participant explained that with time they improved in the role of facilitating the small group learning process. Specifically, the participant stated, “I am always working on the group dynamics. I am getting better at teaching students how to work in groups.” These and other comments indicated that a diverse group of facilitators viewed the same roles as a challenge.

In general, facilitators indicated that they face a number of challenges, many of which describe the discomfort with their role. The level of discomfort seemed to vary from facilitator to facilitator. Time, curricular, and resource constraints were also perceived as challenges, though of a lesser magnitude than facilitators’ discomfort with their roles. The open response question on the perception of facilitators’ PBL implementation challenges suggests that challenges vary among participants with the roles of the facilitator mentioned most often as a significant challenge. The responses from this open-ended question seemed to support the results from the closed response questions.

Implications

With minor revisions, the *Facilitator Perception Survey-Revised* may be useful in other secondary education content areas using the PBL approach. It is applicable for new facilitators, practicing facilitators, individuals considering PBL facilitation, and researchers. The survey and the findings of this research also have practical implications for teacher educators involved with teacher development in secondary education. Barrows (1994) argued that the training of the PBL facilitator impacts the success or failure of PBL. Individuals designing learning experiences for facilitator training workshops may find this instrument useful as a vehicle to examine facilitators’ perceptions. It may serve as a tool to engage workshop participants in activities to stimulate discussion on PBL issues. It can also be used as a self-evaluation tool. This may also increase self-awareness of a commitment to the PBL model that may benefit professional development programs and individuals.

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LISTENING AND QUESTIONING STUDENTS' MATHEMATICAL THINKING

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In this pilot study, listening and questioning practices of a secondary mathematics teacher were investigated for two lessons within small group and independent practice time. The two lessons included a traditionally structured lesson on dividing decimals and an inquiry-based lesson on systems of linear inequalities. Using thematic analysis, the types of questions asked, questioning patterns, and modes of teacher listening were deductively coded, and questioning sequences were inductively analyzed to characterize how teachers listen to and question students' mathematical thinking. Findings included four themes, two for each lesson, which indicated a transformation of the teacher's listening and questioning practices.

In recent years, mathematics education researchers have become increasingly interested in professional noticing of students' mathematical thinking (Jacobs, Lamb, and Philipp, 2010; Sherin & van Es 2009). Davis (1996) proposed a "sound alternative" (p. xxi) to visually-focused mathematics teaching practices by attending to teacher listening. Echoing Davis, this pilot study complements professional noticing by attuning to teacher listening as well as associated teacher questioning practices implemented in secondary mathematics classrooms.

Because the act of noticing is a hidden practice (Jacobs et al., 2010) involving attention to nonverbal artifacts, this paper used the term monitoring (Smith & Stein, 2010) as a broader teaching practice. In the act of monitoring, a teacher might notice students' written work and body language, and also teachers have the opportunity to listen to student thinking during small group work. The purpose of this pilot study was to characterize how teachers listen to and question students' mathematical thinking while monitoring student work. The research question addressed: What are characteristics of a mathematics teacher questioning patterns when a teacher listens to students' mathematical responses while actively monitoring small group work?

Related Literature

Question Types

National Council of Teachers of Mathematics (NCTM, 2014) stressed the importance of the types of questions teachers ask. NCTM's framework included four question types: gathering information (what do students know?), probing thinking (can students explain their thinking?), making the mathematics visible (can students relate mathematical ideas?), and encouraging justification and reflection (can students prove their strategy works in other problems?). In typical traditional lessons, teachers rely on gathering information questions providing students "very little opportunity to engage in meaningful mathematical activity" (Wood, 1998, p. 172). To identify teacher questions, Boaler and Brodie (2004) suggested including statements functioning as questions (e.g., The linear parent function is-) since some questions do not seek a response.

Questioning Patterns

Eddy, Harrell, and Heitz (2017) reasoned that "for teachers to be effective questioners they need both quality questions and good question structure" (p. 11). Traditional questioning structures have been associated with Mehan's (1979) initiate-response-evaluate (IRE) sequence. A teacher initiates a sequence with a gathering information question. Then, the teacher ends with an evaluative move indicating correctness. IRE structures funnel student attention towards a predetermined response (Wood, 1998). Reform-oriented revisions of the IRE sequence include the initiate-response-follow-up (IRF) sequence to indicate a non-evaluative move and the expanded IRFRF format (Chin, 2006). In an IRFRF, student responses and teacher follow-up moves repeat until a final teacher response (i.e., "okay," "very good"). Follow-up moves could be evaluative in a funneling pattern. However, when teacher questions attune student attention towards connections of student ideas, these are focusing patterns (Wood, 1998).

Modes of Listening

Davis (1996) conceptualized three modes of listening (evaluative, interpretive, and hermeneutic) to describe ways in which a middle grades teacher enacted listening. Evaluative listening positions a teacher in an authoritative role seeking correctness in student responses, whereas interpretive listening opens dialogical spaces for student sense-making. However, Davis noted that mathematical authority remained with the teacher during interpretive listening since student interpretations were paralleled to teacher explanations. Furthermore, when enacting interpretive listening during a discussion, the teacher maintains authority over which student ideas to amplify. To that end, Davis described a third type, hermeneutic listening, as a mode of listening in which the teacher enacts a participatory role with the collective learners.

Wait Time

Rowe (1986) acknowledged teacher wait time contributed to in-depth student responses. Rowe claimed that when teachers wait at least three to five seconds after a teacher question (WT1) as well as after student responses (WT2) (see Figure 1), the extra student-thinking time afforded students opportunities to elaborate on thoughts and increased student-to-student interactions. Both situations create opportunities for teachers to listen to students' mathematical reasoning.

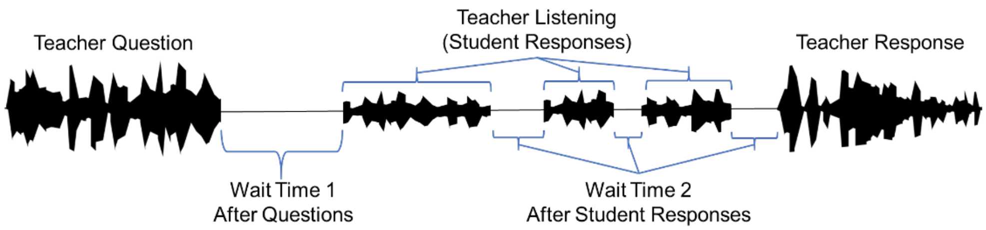

Figure 1. IRE soundwave image. IRFRF responses include chained images of the IRE figure. Image adapted from https://openclipart.org/detail/202430/raseone-soundwave-1.

Method

Participant

One teacher, Sharon, was a veteran secondary teacher teaching in an urban school district. She was purposefully selected for this pilot case study because her questioning had improved from the first observation to second observation. During the first classroom observation, she taught a traditional lesson as she modeled, guided, and monitored individual practice on decimal division problems. The second observation involved an inquiry-based lesson as students investigated systems of linear inequalities in small groups.

Data Collection and Analysis

Sharon's two classroom observations were video recorded using Swivl™ (Tetelbaum & Lamb, 2010) robot. Thematic analysis (Braun & Clarke, 2006) included data preparation, deductive provisional coding, and inductive tabletop categorization (Saldaña, 2016). Coding was performed for the 7.5-minute sections of small group or independent practice (see Figure 2) only because that is when the teacher had opportunities to listen to student thinking while monitoring student work. For this study, the last 15 minutes of each lesson were coded for comparison.

	0:00-7:30	7:30-15:00	15:00-22:30	22:30-30:00	30:00-37:30	37:30-45:00
Lesson 1	Whole Group	Whole Group	Whole Group	Whole Group	Independent	Independent
Lesson 2	Small Group	Whole Group	Small Group	Small Group	Small Group	Whole Group

Figure 2. Lesson map indicating audience level for more than half of each 7.5-minute section.

Data preparation. During data preparation, both observation videos were watched and re-watched to gain familiarity with the lesson flow. The transcribed videos were focused on teacher talk-turns but excluding student talk-turns since student information was not included in the IRB. Transcripts indicated teacher pauses greater than three seconds in place of student talk-turns.

Next, each 45-minute lesson was divided into 7.5-minute sections (Hill, Charalambous, & Kraft, 2012) to map when small group or independent practice time occurred for at least half of a section (see Figure 2).

Two researchers independently used V-Note® (Emig, 2014) to code teacher questions, wait time, teacher listening, monitoring, teacher explaining, general teacher responses, and non-instructional time. Monitoring occurred before teacher questions, while wait time occurred after teacher questions and between student responses. The teacher's responses to students were coded as teacher explaining of mathematical ideas or as a general teacher response (e.g., that's right!). Non-instructional time included administrative tasks or classroom management.

At the outset of this project, the times of the teacher monitoring, explaining, responding, and non-instructional time were not considered as separate identification markers. However, after reviewing the videos, both researchers agreed that identifying each instance aided in the purposeful selection of lesson sections for further analysis. Intercoder agreement was attained through regular conversations throughout the data preparation phase.

Provisional coding. Provisional coding (Saldaña, 2016) was used as a theory-driven approach, beginning with established frameworks and allowing for updating of codes during the coding process. Using V-Note®, NCTM's (2014) framework for types of questions and Davis's (1996) modes of listening were used to initially code question types and modes of listening. In Figure 3, the questioning and listening frameworks are interrelated in one matrix.

Mode of Listening / **Question Type**	Evaluative (for)	Interpretive (to)	Hermeneutic (with)
Gathering Information (know)	What do you know about inequalities?		
Probing Thinking (explain)	How did you know which way to shade?	How did the coordinate plane help in this task?	I'm not sure how you found the inequality, can you show or explain to me a different way?
Making the Mathematics Visible (relate)	Based on the graph, what's the difference between an equation and an inequality?	Explain the difference between shading and not shading the streets on the map.	How is this task different or the same as the first task?
Encouraging Justification and Reflection (prove)	How do you know that you found the mystery location?	How might you prove that you have found the mystery location?	Design a scenario using three inequalities and your own mystery location.

Figure 3. NCTM's (2014) framework for questions used in mathematics teaching and Davis's (1996) modes of listening combined. Questions in the hermeneutic listening column were created for the matrix as Sharon's lessons did not include questions that invoked hermeneutic listening.

Two cells are gray because when one asks gathering information questions, the intent is to listen for specific responses. In the matrix, example questions were adapted from Sharon's second lesson except for the hermeneutic listening column. These questions were created since hermeneutic listening of mathematical thinking was not evident in either lesson.

In addition to mathematical questions, general questions and soliciting student responses codes were added. General questions included non-functioning questions (e.g., Right?) (Boaler & Brodie, 2004), classroom management, or progress management (e.g., Are you done yet?). Teacher listening was not coded when student responses were not mathematical. Example questions included when Sharon called on a student, asking students to repeat their response (e.g., Did you say something?) or seeking student questions (e.g., What's your question?).

Categories. To prepare for an inductive analysis, IRE and IRFRF lesson segments were identified within the 7.5-minute sections. Lesson segments were partitioned based on initiating teacher questions (see Figure 3) and final teacher statements. Each lesson segment represented one teacher-student interaction. Last, the V-Note® coding display (see Figure 4) and transcripts were printed and physically sorted on a tabletop to identify recurring patterns (Saldaña, 2016). For both lessons, lesson segments were selected from the last 15 minutes (two 7.5-minute sections) of each lesson for a direct comparison. For the first lesson, 8.5 minutes were spent in independent practice across the last two lesson sections, while in the second lesson 7 minutes and 47 seconds were used for small group instruction across the last two sections. The remainder of the 15 minutes were spent in whole group instruction.

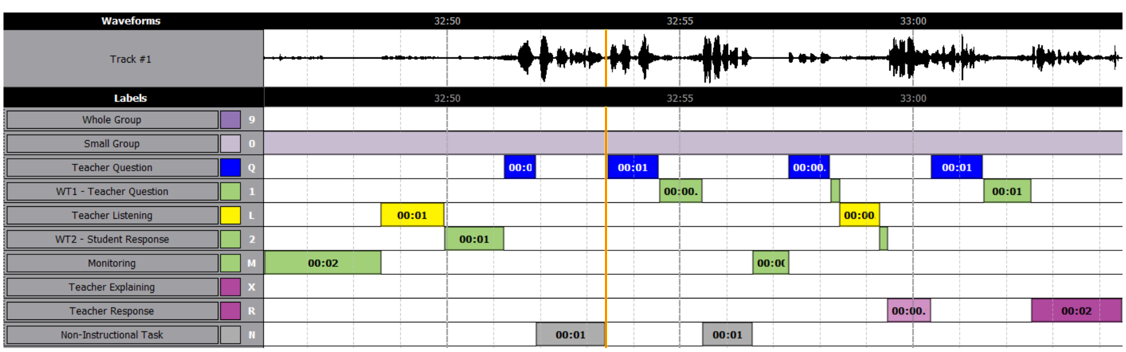

Figure 4. A portion of the V-Note® interface displaying data preparation codes and the soundwave image.

Findings

Four categories (two within each lesson) were identified during the tabletop sorting. Table 1 summarizes the instructional time spent during the last fifteen minutes of each lesson. Wait time after teacher questions and after student responses were separated in the wait time column.

Table 1

Duration and Percentage of Instructional Time

Lesson Type	Monitoring	Questioning	Listening	Wait Time	Explaining	General
Traditional (8:30)[*]	3:11 (37%)[+]	0:30 (6%)	0:16 (3%)	0:17 (3%)	0:59 (12%)	0:20 (4%)
Dividing Decimals				0:12 (2%)		
Inquiry-based (7:47)[**]	0:45 (10%)[++]	0:55 (12%)	0:28 (6%)	0:19 (4%)	1:07 (14%)	0:50(11%)
Systems of Linear Inequalities				0:12 (3%)		

Note: [*]The total time spent in independent practice.[+]Percents in this row are out of 8:30. [**]The total time spent in small groups.[++]Percents in this row are out of 7:47. Non-instructional time accounts for the remainder of the small group and independent practice time in each row. These measures were excluded from analysis as they were beyond the scope of the present study.

Lesson 1: Traditional Lesson on Dividing Decimals

Monitoring student work. Students worked independently at their desks with no student-student interactions as Sharon walked amongst students working. The lack of interactions provided minimal opportunities for her to listen to student understandings, and interactions with only seven students were observed. Long stretches of monitoring time occurred between each interaction. Two students were asked if they wanted to display their work on the board; one agreed. Initiating questions for the next three interactions began with a general question to check progress (e.g., How’s it going?; Do you get this?). After asking the fifth student if he had a question, Sharon had the opportunity to listen to his understandings. The lesson segment ended as she explained how many decimal places to move. The last interaction started with a student question followed by funneled IRFRF sequence. Sharon ended the sequence by asking “How many groups of 11.86 [in 40]?” before moving to the next student without waiting for a response.

Teacher explaining. Sharon asked a total of 17 questions spending only 30 seconds on questioning, and double that time explaining. Out of the 17 questions, six questions were the gathering information type. One gathering information question (Will 18 divide into 37?) was posed to a non-responding student before Sharon moved to addressing the whole group. The remaining five gathering information questions were in the IRFRF sequence (e.g., How many 11s are in this 40?). In all instances, Sharon evaluatively listened for specific responses, asked questions in a funneling pattern, and explained how many places to move the decimal. She spent the most time correcting the student at the board. Rather than using the student mistakes as a learning opportunity, Sharon explained each step before moving to the whole group.

Lesson 2: Inquiry-Based Lesson on Systems of Linear Inequalities

The goal of this lesson was for students to find a mystery location using systems of linear inequalities. Clues were provided, and students worked in small groups of three to four students to determine where the mystery location could be located. Sharon interacted with five different student groups, returning several times throughout the coded lesson sections.

Focusing to funneling questioning pattern. In several lesson segments, Sharon adjusted the questioning pattern from focusing on student ideas to funneling student responses to predetermined endpoints. The lesson segments indicated Sharon's use of interpretive listening as she made sense of student understanding. However, once she was able to gauge a student's learning trajectory, the types of questions asked focused on probing student thinking and gathering information to lead students to her way of thinking. Sharon reverted to repeating and rephrasing questions to elicit predetermined student responses.

Multi-leveled evaluative listening. Multi-levels of evaluative listening were evident in the different types of questions that were asked. For example, while Sharon may have asked, "What do you notice about the shading?", she was still looking for a specific response to the inequality task. Sharon asked a total of 35 questions, which was double that of the first lesson. The types of questions included 14 gathering information questions (e.g., What kind of symbol do I still see in your inequality?), one probing thinking question (Where do you want your [y-intercept]?), one making the mathematics visible (Why is there a coordinate plane on your map?), and the remaining questions were general questions. In contrast to the first lesson, Sharon spent about an equal amount of time asking questions as she did explaining concepts.

Implications

The structure of the lessons contributed to the difference in Sharon's teacher questioning and listening. With the change in lesson structure, Sharon interacted with more students, increased the number of questions asked which in turn increased the time spent listening. Wait time for both lessons were about the same indicating an area of growth. As Rowe (1986) claimed, when teachers attain an average wait time of at least three seconds, the quantity and quality of questions also improve. Teachers, such as Sharon, who are provided opportunities to practice intentional listening within supportive professional development, are more likely to attune teacher questions to student ideas presented in-the-moment of teaching.

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EXAMINING LONGITUDINAL OUTCOMES OF BLENDED PROFESSIONAL LEARNING

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This research study investigated the long-term outcomes of an innovative blended professional learning program designed to transform elementary teachers' approaches to mathematics teaching and learning in accordance with recently adopted content and practice standards. This paper describes the professional learning and its sustained impact by examining the ways in which participants continued to implement their acquired knowledge eighteen months after the program's conclusion. Site visits to seven of the project's 15 partner districts resulted in 28 teacher interviews and 25 classroom observations. The reported findings chronicle positive outcomes related to classroom practice, collaboration, and teacher leadership.

Introduction

As mathematics educators, we desire all teachers to be life-long learners. This desire fuels our efforts to construct professional learning opportunities enabling teachers to thrive and grow in their profession. Effective professional development is founded on a multi-dimensional, long-term framework that integrates meaningful content, active learning, and collective participation (Desimone, 2009). It equips teachers for continuous growth, for enhancing student achievement, and for leading others (NCSM, 2008). One way to incorporate these qualities into extended professional learning is through a blended approach combining face-to-face and online methodologies. This study investigated the longitudinal impact of such a program.

The STREAM project (Standards-based Teaching Renewing Educators Across Montana) was funded as a three-year U. S. Department of Education Mathematics and Science Partnership to "provide intensive, content-rich professional development to teachers and other educators, with the goal of improving classroom instruction and ultimately, student achievement in math and science" (U.S. DOE, n.d.). The project was guided by a mandate to (1) create a statewide systemic, research-based and sustainable approach for improving teacher content knowledge and student achievement of Montana's adopted version of the Common Core State Standards for mathematics, and (2) design and deliver interactive, on-demand, high-quality learning modules for statewide professional development via school-based and distance learning.

The STREAM partnership included faculty at two universities working with teams of teachers and administrators from 15 school districts of varied size and demographic makeup. Two cohorts of teachers in grades 4-7 participated in blended professional learning designed to increase their own understanding of mathematics content and mathematical practices, while preparing them to effectively implement the new standards and share their expertise with peers. Each cohort completed eight months of blended professional learning with face-to-face and online components, then composed strategic plans for implementation and dissemination in their districts the following year. Pre-post test results confirm that the STREAM program increased participants' knowledge of mathematics content and practice (e.g., large effect size with significant gains for 84.5% teachers in the first cohort; for detailed results see Luebeck, Roscoe, Cobbs, Diemert, & Scott 2017; Shaw, 2013, 2014). Evaluation data show that upon completing the program teachers were highly motivated to apply their learning; however, the project did not have funding to follow teachers back to their districts and observe the effects of STREAM professional learning on their classrooms and colleagues.

This research study, conducted 18 months after the project's conclusion, explored what elements of the "STREAM experience" were enduring, as well as how they influenced teacher practices and student learning. Concisely stated, the research questions guiding this study are: How did the STREAM project's professional learning design impact teaching and learning in mathematics classrooms? How did participating teachers experience and describe this impact? Answers to these questions illuminate the long-term outcomes of STREAM professional learning, offering implications for similar programs and further research.

Framework

Professional development has long been viewed as a primary vehicle for teacher change and improvement. In 1991, the National Council of Teachers of Mathematics called for the design of "pre-service and continuing education programs that reflect the issues of reform and change that must be implemented" (p.184). Twenty years later, the Conference Board of the Mathematical Sciences (CBMS) affirmed that the "preparation and further education of mathematics teachers" is a widely shared responsibility whose "collective goal needs to be continual improvement" (2012, p.3). Effective professional development is grounded in theories of adult learning (Merriam, 2001) and transformational learning (Baumgartner, 2001). It is also grounded in the content needs of teachers. The CBMS observed that content-based professional learning supported by federal math-science partnerships for K-12 teachers has "changed their attitudes about mathematics, and increased their mathematical interest and abilities. Moreover, it has increased the achievement of their students" (2012, p. xii).

A focus on content is only one of several criteria shown to support effective professional development. These include: (1) alignment with school goals, state and district standards and assessments, and other professional-learning activities; (2) a focus on core content and modeling of teaching strategies for that content; (3) opportunities for teachers to engage in active learning; (4) provision for sustained and collaborative learning; and (5) follow-up and continuous feedback (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; DeMonte, 2013; Reiser, 2013). These criteria parallel the components of Desimone's "core conceptual framework" for effective professional learning (2009, p. 183), which advocates for a focus on content, active learning, coherence, duration, and collective participation. The STREAM project made a deliberate effort to address these criteria in its blended professional learning design.

STREAM Professional Learning

The blended approach to STREAM professional learning combined four online modules with three face-to-face workshops. The curriculum emphasized Mathematical Practices in the context of Number Systems/Operations and Ratio/Proportion, both essential content domains in grades 4-7. A theme of Teacher Learning and Leadership prepared participants to implement and share their learning upon returning to their districts. These four themes (Figure 1) were launched at a workshop where teachers learned mathematics, examined standards and strategies, and prepared for subsequent online learning. After completing two online modules, they met again midyear to engage in active and collaborative learning designed to form connections across the themes.

Theme 1 - Common Core Mathematical Practices (MPs) and STEM Connections
Teachers learn how to embed MPs in everyday instruction, how to assess students' use of MPs, and how modeling and other MPs provide natural connections to science, technology, and engineering.
Theme 2 - Grades 4-7 Learning Progression: Number Systems and Operations
Teachers learn how concepts progress and unfold across clusters, domains, and grade levels. They trace the development of number concepts through operations, properties, and systems.
Theme 3 - Grades 4-7 Learning Progression: Fraction-Ratio-Proportion
Teachers learn how concepts progress and unfold across clusters, domains, and grade levels. They trace the development of proportional reasoning to its conceptual roots in fraction and ratio.
Theme 4 - Teacher Learning and Leadership: Facilitating PLCs, Modeling Instruction
Teachers are expected to disseminate materials from Themes 1-3 in their home districts. Theme 4 provides them with skills and strategies to promote school-based collaborative teacher learning and to model standards-based instruction for their peers.

Figure 1. Outline of the STREAM project's thematic curriculum.

Following the final two online modules, a four-day summer academy provided opportunities for teachers to work on extended mathematics tasks, synthesize their learning, and develop strategic plans to productively apply their knowledge in the coming year. Each district team filled out a strategic planning template where they described in detail their goals, the scope of their implementation plans, and their intended audience. They prepared a sequence of specific professional learning activities to occur within a one-year timeline. After two cycles of review and revision, the plans were put into action in teachers' home districts the following academic year. The first cohort of teachers (N = 39) carried out their strategic plans over two years; the second cohort (N = 19) enacted their plans for only one year before the three-year program was concluded. Reports submitted by district teams during the implementation period showed that the strategic plans were, for the most part, carried out as intended. This research study focused on sustained outcomes beyond the funding period, examining how participants continued to implement their acquired knowledge many months later (Table 1).

Table 1

STREAM Project Research Timeline

Year	Participants	Research-Related Activities
Y1: 2012-13	39 teachers (15 districts)	Y1 teachers engage in professional learning
Y2: 2013-14	19 teachers (7 districts)	Y1 teachers implement district strategic plans Y2 teachers engage in professional learning
Y3: 2014-15		Y1 teachers continue district strategic plans Y2 teachers implement district strategic plans
Y4: 2015-16	Ongoing individual and district implementation – no project monitoring	
Y5: 2016-17	28 selected teachers	Data collection: school site visits and interviews

Methodology

Data were collected from the Year 1 cohort, comprised of 39 classroom teachers from 15 district teams, as well as project mentors and administrators. Employing purposeful selection, the researchers identified teams that were successful in carrying out their strategic plans and represented the widest possible range of district demographics. The seven selected districts included one independent K-8 district, two remote and rural districts (one with only 35 students), two districts in medium-size cities, and two large districts (one with over 11,000 students). Surprisingly, six of the seven administrators who originally partnered with the project no longer held the same position. Establishing contact was also challenging in a few cases where teachers had left a district or moved into administration. Most responded willingly to a request to participate, leading to site visits and interviews conducted with 28 teacher participants over a 5-week period in Fall 2016.

The original intention was to introduce the interview protocol in an on-site interview and audio record teachers' responses. However, several teachers early in the process asked to see the interview questions in advance and, unsolicited, sent their typed or handwritten responses to the visiting researcher prior to the site visit. This pattern became the norm and proved to be an effective strategy for in-depth data collection. By writing out narrative responses to the protocol, teachers had time to reflect and thus provide more authentic perceptions of how their STREAM experience continued to influence mathematics teaching and learning in the present day. In some cases, they reflected on the questions with other STREAM participants from their school, resulting in more accurate recall and more sensitive observations of impact.

The visiting researcher reviewed the written responses to prepare for each interview, noting opportunities to clarify, expand on, and probe the broadly posed interview questions:

1. In what way has your STREAM experience influenced your teaching?
2. In what way has your STREAM experience influenced your interaction with others (teachers, students, administrators)?
3. What, if any, influence has your STREAM experience had on other aspects of your professional activity?
4. How, if at all, has STREAM influenced the continued implementation of Montana's Common Core Standards for Mathematics in your school?

During the 20- to 40-minute interviews, the researcher scripted detailed notes elaborating on the written responses. Interviewing one participant sometimes resulted in a referral to another STREAM teacher in the school who did not initially respond to requests for an interview. To enhance and provide context for the interviews, the visiting researcher also interviewed current administrators and informally observed mathematics lessons taught by STREAM participants while on site. Written teacher narratives and scripted interview notes were collected from 24 full-time teachers, with additional data gathered from two teacher-coaches and two teacher-administrators. Where possible, informal data from school administrators and classroom observations were used to corroborate data reported in the teachers' narratives and interviews.

The first phase of analysis involved only the visiting researcher who conducted the interviews. Written responses to each interview question were analyzed separately and a set of codes developed for each question. The codes, along with corresponding evidence in the form of quotes, were entered into a spreadsheet. These codes were then augmented with additional data from the scripted interview notes. Codes representing similar concepts were grouped into categories, while others were identified as distinct. In the next phase of analysis, the second researcher reviewed the codes and categories, referring to the raw data for clarification as needed, and either confirmed the uniqueness of a category or identified similar categories that could be combined. Where analysis differed (about 25% of categories), both researchers re-examined the data and codes until they reached consensus. In this process, some categories were collapsed or condensed, while others were enhanced with additional data.

Findings

Teachers described and demonstrated a variety of sustained positive outcomes that they attributed to their STREAM professional learning experience. They also noted outcomes and practices that had diminished over time due to changes in district priorities, administrative turnover, or other circumstances. The findings presented here represent the most powerful sustained outcomes, condensed into three overarching themes and discussed below.

Transformed Instructional Practice

Teachers reported unequivocally that as a result of participating in STREAM, they were consistently applying more research- and standards-based instructional strategies in the classroom. The most frequently reported changes were increased use of hands-on teaching methods, transforming traditional direct instruction into a student-centered approach, and incorporating critical thinking questions. Teachers also noted qualitative changes in teaching: "STREAM…taught me to go deeper with mathematical concepts….We need to have the big picture." One teacher described a shift toward giving students greater ownership of their learning and encouraging peer-to-peer discussion, ending with "I became a constructivist!"

Teachers also reported that they continued to purposefully incorporate the Common Core Mathematical Practices into instruction, due largely to how this was modeled in the STREAM curriculum. They articulated the importance of deliberately engaging their students in well-defined and appropriate mathematical practices on a consistent basis – not merely posting the list of practices on a classroom wall or vaguely referencing them in lesson plans.

Collaboration and Community

A sustained pattern of professional collaboration and personal interaction has resulted from participation in STREAM professional learning. One teacher valued the strong bonds formed through "building relationships and working deeply over time." Lesson studies and peer observations, conducted within schools or districts as part of STREAM strategic plans, opened up levels of communication that had not existed before. Several referred to an increase in the "amount of discourse among staff members," particularly across multiple grades. The largest district team noted that "teachers share across grade levels," while in the smallest district, "STREAM got us to do K-8 things together." Teachers at one school described how STREAM motivated them to share their students' progress at the end of each year and discuss challenges students might have as they entered the next grade. Another teacher echoed: "Our STREAM goals helped encourage common expectations and language in math."

Teacher Confidence and Leadership

Participants repeatedly referred to how the STREAM experience increased their confidence – not only in their own understanding of mathematics, but also regarding how to effectively teach mathematics. This propelled them to collaborate productively with other teachers and to facilitate learning among their peers. A teacher from a large district observed, "The STREAM members are still viewed as math leaders in their schools…helping make decisions that will move the district forward." Another added, "We are still used as a resource in our building."

A number of STREAM participants had moved beyond self-improvement to leading others in their school and district or taking on more systemic leadership roles (NCSM, 2008). One of these claimed that prior to her STREAM experience she would never have viewed herself as a mathematics educator. She and other participants reported how they were influencing mathematics education across Montana through modeling effective instruction, providing professional development, or working on task forces and projects for the state education office.

Implications and Lessons Learned

This study investigated the enduring outcomes of a program built on evidence-based criteria for effective professional learning. It validates the potential of blended learning as a vehicle for delivering high-quality professional learning with sustained positive effects over time. Building on the STREAM curriculum framework, future studies should explore how specific elements of professional learning generate the sustained outcomes observed in this study: transformed instruction, community building, and leadership development. This research provides a foundation to examine the efficacy of blended and online learning in other settings, particularly where barriers of distance, expense, or time limit opportunities for traditional professional development. Studies could also address challenges experienced in the STREAM project and inherent to blended learning. How does a program serving multiple districts and grade levels calibrate the content and delivery of professional learning to accommodate wide variations in participants' mathematical knowledge? How do developers balance respect for teachers' limited time and competing priorities against a commitment to rigorous professional learning?

"Three years is not enough!" and "No, it can't be over!" were two sentiments voiced by teachers upon completion of the STREAM project. This research study confirmed that effective professional learning need not be "over" – it can produce lasting effects on teacher practice. The participants in this study experienced, embraced, and now continue to implement standards-based mathematics content and evidence-based teaching strategies that engage their students in mathematical practices. They developed leadership skills that have visibly improved communication and collaboration among mathematics teachers in their districts, and they are exemplars for the next generation of mathematics teachers. Positive outcomes such as these call for continued study of effective models for teacher professional learning.

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MATHEMATICS LESSON STUDY: TEAM-CREATION FRAMEWORK

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The strength of collaborative approaches in staff development lies in opportunities for teachers to productively work in a positive and healthy environment. This qualitative study examined how forming collaborative learning teams based on content knowledge, algebra teacher self-efficacy, and teacher preference shaped the interpersonal dynamics of mathematics lesson study teams. The results from teacher reflections suggested the team-creation framework encouraged positive and productive interpersonal team behaviors. This framework can inform the creation of lesson-study teams as well as other professional development programs that use collaboration.

For over five decades, public schools have been subjected to increased accountability as state and federal governing bodies have become more involved in creating systems to improve the outcomes of public education (Spring, 2014). Because of this increase in accountability, many schools are implementing a variety of professional development programs (PD) in order to enhance teacher learning to impact student achievement. One such PD to enhance teacher learning is lesson study (LS). This is a collaborative approach in which teachers engage in a process that improves the quality and effectiveness of their teaching (Lewis & Hurd, 2011; Isoda, 2015). This involves a small team of teachers who work together to design, observe, critique, and redesign lessons. The key to LS is the successful interactions among teachers. If learning is interrupted by dysfunctional interpersonal relationships, such as unproductive conflict, the fundamental goal of collaborative learning can be diminished (Achinstein, 2002).

While lesson study teams have been created by campuses or academic subjects, no research to date has investigated purposeful formation of lesson study teams. As a result, the following research question was investigated: How does the purposeful formation of mathematics lesson study teams shape interpersonal team dynamics? For the purpose of this study, a *team* refers to a collection of teachers who have a common learning goal who work interdependently and hold each other accountable. On the contrary, a *group* is a loose collection of teachers who lack the specific aforementioned characteristics of a team (DuFour, DuFour & Eaker, 2008).

Theoretical Framework

In order to understand how the interaction of individuals within a team influences performance, the researchers drew upon Belbin's (2007) role theory as a useful framework. He explained teams who included members with a wide range of roles performed better than teams with less role variety. While nine specific roles were identified, he also espoused that the foundation for successful interaction rested on a balance between *functional* and *team* roles. Functional roles are associated with individuals who possess technical and/or professional knowledge. These individuals help focus the team on the assigned task, while *team* roles are associated with team members who exhibit positive interpersonal behaviors. The *team* role can act as a mediator who promotes team cohesion. Ultimately, the premise of Belbin's (2007) theory hinged on the idea of balancing these two roles within a team.

Prichard and Stanton (1999) reinforce the importance of these roles by describing that both task process (*functional* role) and maintenance skills (*team* role) are fundamentally important to successful team interactions. Task process refers to accomplishing the team's goal by defining the problem, collecting information, and developing a solution. Maintenance skills refers to the social or emotional interactions between team members that maintain team processes. Task process and maintenance skills are analogous to *functional* and *team* roles as articulated by Belbin (2007). We contend that team members with higher levels of content knowledge would perform the task process (*functional*) role, while members with high teacher self-efficacy would support the maintenance skill (*team*) role. Both are critical to team performance.

For the purpose of this study, algebra teacher self-efficacy (Wilkerson et al., 2018) served as the *team* role in order to promote harmony in the team. Teachers with elevated levels of teacher self-efficacy are associated with high commitment, confidence, motivation, resilience, and goal setting (Bandura, 1993). They are less likely to compare themselves to others which could diminish the likelihood for conflict and power struggles. Teachers with content knowledge (Prichard & Stanton, 1999) served the *functional* role as they possess the technical and in-depth knowledge of mathematics. Because studies have shown that allowing individuals to select their own teams has a positive effect on perceived team experiences (Chapman & Van Auken, 2001), this factor was included in the team-creation framework.

Methods

Of the 17 teachers who voluntarily participated in this study, 13 had been together the first year of a two-year eighth grade mathematics and algebra PD. An additional four teachers were added the second year who either worked with or knew one of the existing teachers in the program. This program consisted of 99 hours of mathematics instruction (60 summer hours focused on algebra content and 39 hours focused on teaching algebra during the academic year) focused on incorporating formative assessment strategies and inquiry approaches to teaching eighth grade mathematics and algebra. The teachers used the Lesson Study with Open Approach (LSOA) model to facilitate their professional learning (Isoda, 2015).

The team-creation framework was based on measures of content knowledge, algebra teacher self-efficacy, and teacher preference. For content knowledge assessment, algebra teachers completed the Teacher Content Knowledge Survey (Tchoshanov, 2011), and eighth grade teachers completed the Diagnostic Teacher Assessment for Mathematics and Science (Saderholm, Ronau, Brown, & Collins, 2010) for algebraic reasoning in the middle grades. Teacher self-efficacy was measured using the Algebra Teacher's Self-Efficacy Instrument (ATSEI) survey (Wilkerson et al., in press), and a teacher preference form was created by the authors and given at the beginning of the second week of the summer. This form allowed teachers to list individuals with whom they preferred to work, and if necessary, one individual whom they believed would inhibit their professional growth in a team setting. This allowed space to list someone whom they believed would inhibit their capacity to function within a team. There were a total of eight participants who made this request and all were honored; therefore, the number of people working with someone identified as a potential conflict was zero.

The Jenks Natural Breaks Optimization Process (Jenks, 1967) was used for ranking team members from the highest Class 4 to the lowest Class 1 for both the mathematics content knowledge and algebra teacher self-efficacy. This created three teams of four and one team of five. After examining the last factor of preference, teams were rearranged (See Table 1) so each teacher had at least one person they selected while maintaining balance in *team* and *functional* roles. Pseudonyms were used for the teachers to maintain anonymity. The goal was to achieve as much balance as possible using team means to determine the level of each teaming factor.

Table 1

Final Teaming of Math Teachers

	Content Knowledge			Knowledge Efficacy in Algebra			Personal Teaching Efficacy in Algebra			Number of Preference Matches	Number of Conflict Matches
	Jenks Class	Score Mean	Team Mean	Jenks Class	Score Mean	Team Mean	Jenks Class	Score Mean	Team Mean		
Team 1			52%			73%			66%		
Aria	4	85%		4	88%		2	56%		No Pref.	0
Paula	1	8%		2	65%		3	65%		2	0
Lisa	3	65%		2	70%		3	69%		2	0

Randall	2	50%		2	70%		3	70%		2	0
Team 2			62%			72%			57%		
Marie	2	55%		3	77%		2	48%		1	0
Sharon	2	50%		1	31%		1	37%		1	0
Blake	4	85%		4	96%		4	78%		2	0
Chloe	3	58%		3	82%		3	63%		1	0
Team 3			55%			74%			65%		
Elena	4	80%		3	74%		2	57%		1	0
Juanita	2	40%		2	59%		3	63%		1	0
Henry	3	60%		4	85%		4	84%		No Pref.	0
Natasha	2	40%		3	78%		2	54%		1	0
Team 4			47%			68%			63%		
Kamace	3	68%		2	70%		3	68%		2	0
Ofelia	1	20%		1	23%		1	21%		2	0
Fatima	2	35%		4	95%		4	92%		1	0
Deanne	3	60%		3	75%		3	65%		2	0
Gino	2	50%		3	77%		3	70%		1	0

Once the teams were created using the team-creation framework, participants conducted a round of LSOA as part of the PD that began in the summer. As the teams conducted LSOA, the researchers utilized the formation of the teams to gain an understanding of how teachers experience their subjective meanings related to professional learning in a social context. A phenomenological methodology was used to guide this study (Hesse-Biber, 2017), and data were collected through reflective journal entries (Table 2) of 17 secondary public-school teachers. A grounded theory coding approach was used to develop themes associated with the research question (Charmaz, 2004). Note, Henry was not available to respond to the September prompt, and Juanita was not available to respond to the October prompt thus reducing the number of responses to 16 for both months.

Table 2

Questions and Dates for Reflective Journal Entries

Date	Reflective Prompt	# of Responses
September	Describe how you "get along" with the other teachers in your lesson study team. How does this relationship help or hurt your learning?	16
October	Describe the role you assumed during your work in the lesson study team. For example, did you assume the role of a leader, a mediator, math specialist, etc.? Explain the process of how members in your team developed certain roles.	16

Findings

Themes and patterns were developed from the teachers' reflective journal entries, then initial memos were drafted followed by initial coding by assigned categorical codes. This allowed the construction of data coded and grouped by general categories of meaning. In response to the research question, the grounded theory process (Charmaz, 2004) was used to reveal two predominant themes. As patterns and themes emerged from the reflective journal entries, analytical codes were assigned to capture a wider range of meaning related to how teachers perceived their interpersonal team dynamics. The reflective journal entries were reviewed until theoretical saturation was achieved; thus, no more useful information was obtained from reviewing the data. The first theme was positive interpersonal team dynamics and the second was an understanding that positive relationships were important to quality collaborative learning.

Teachers reported experiencing positive interpersonal team dynamics. Out of the 16 responses, 13 indicated experiencing positive team dynamics with three providing neutral (did not express positive or negative dynamics) results. They reported getting along well with other members of their team and appreciated the positive interpersonal interactions. This dynamic was specifically articulated by Kamace who stated:

> This new lesson study team has a greater personality match than in my previous year's team in which there were a lot of alpha personalities. Given the personalities of my current team, we are not all alphas and it generates a greater balance of leadership, peace keeper, manager, and support that works more effectively.

Kamace received the highest score on the algebra teacher self-efficacy survey compared to the 16 other teachers. This supports the idea that teachers demonstrating high levels of teacher self-efficacy have positive outlooks, high levels of resilience, and a strong commitment to achieving goals (Bandura, 1993). They also believe that skills are acquirable and recover quickly after experiencing failures or setbacks. We suggest these qualities have the capacity to act as motivating factors when working cooperatively with other teachers and can facilitate *team* roles by promoting team cohesion.

This teacher's detailed awareness of team roles and interactions supports the idea that they are capable of acting in a role that creates harmony and good relations among members. This statement also exemplifies the importance of creating teams that include a diversity of personalities. Prichard and Stanton's (1999) research supports this idea as they concluded that the effectiveness of a team will be promoted by the extent to which members correctly recognize and adjust themselves to the relative strengths within the team, both in expertise and ability to engage in specific *team* roles. As exemplified in Kamace's statement, this process of adjustment is much easier when there is a balance as opposed to having too many *alpha personalities*.

Although a clear majority of teachers reported experiencing positive team dynamics, team four indicated there was some tension. For example, Ofelia stated,

> We have had a slow start. We have a first-year member in the team whom has missed some relevant time, and it has impeded our planning time because there are a ton of questions and it has prevented us from being further along in our LSOA planning.

Upon further examination of this team's responses, four out of five noted they “got along” with each other, and one indicated a neutral response. All members did, however, express concern that communication and a slow start was causing issues. As reinforced by Gino, “We get along fine, but communication is a bit lacking.” This stress was also noted by Deanne who reported, “I am concerned about the lesson study next month because from what I know...we do not have anything done.” When examining the responses from the second reflective prompt, this team appeared to have overcome their slow start and were performing well as a team. Gino stated, “It started off a little rocky, but it was pretty cool how it all worked out.” This suggests the team experienced cohesion and positive team dynamics once the issues of communication and procrastination were ameliorated. Tension during team formation is a normal process of team development as long as it does not diminish cohesion (Wheelan, 2005).

The second theme focused on the importance of relationships. Out of 16 responses, 14 supported this theme, while two reported neutral perspectives. The teachers articulated their understanding that positive interpersonal relationships greatly affected the quality of learning in a cooperative setting. Chloe stated:

> I get along with the teachers in my lesson study team. I was a little nervous when assigned teams because I wanted to have a team with people that I can get along with and have a good collaborative experience. I think this helps my learning because I am more open and can talk with people I get along with easier. I have seen my students shut down when they are working with somebody they don’t know at all or have had a previous disagreement with. I think the same concepts would apply with adults.

Her response was similar to others in that they appreciated the cooperative learning opportunities and valued the information gained from peer interactions. They also expressed a certain level of confidence in their team’s abilities based on their perceived positive interpersonal dynamics.

According to Wheelan (2005), as individuals come together in teams, they bring with them a cultural blueprint which informs their values, beliefs, and behavioral norms. When the members have similar blueprints, they initially have higher levels of interaction, cooperation, and productivity because they have similar beliefs and attitudes that can contribute to interpersonal attraction/cohesion between people. When newly formed teams have a large amount of variety in their cultural background, they could have more intensity during the conflict phase of team development as they work to develop common values and beliefs. The expression of nervousness about their assigned team could be an expression of anxiety about developing cohesion with people who have different blueprints. This supports our team-creation framework which provides teachers some choice in their teaming assignments. While it was clearly stated that choice would be limited (priority was placed on content knowledge and algebra teacher self-efficacy), every participant was placed in a team that included one or two of their preferences. The researchers also honored any requests made by a participant to avoid being teamed with a specific individual in order to promote cohesion.

Implications

The results of this study thus far demonstrate the team-creation framework, centered on content knowledge, algebra teacher self-efficacy, and preference, can positively shape the perceptions of teachers as they participate in collaborative PD. This suggests that teachers competent in their content knowledge could be performing their *functional* role, and efficacious teachers could be performing their *team* role by facilitating diplomatic interactions. This indicates that mixing roles using the Jenks Natural Breaks Optimization Process (Jenks, 1967), as well as limited teacher preference, supports Belbin's (2007) role theory. As more data are collected, these implications will be reexamined.

The results of this study were not designed to be generalized; however, the team-creation framework could be deductively examined using quantitative methods. Future research could examine correlations between the three factors and their outcomes on teacher staff development centered on lesson study. This has the potential to greatly impact the field of professional staff development not only in mathematics, but also impact many other content areas as well. As state and federal accountability reforms continue to increase, equipping teachers and administrators to effectively learn and improve will be critical to student, teacher, and ultimately school success. Future studies could also include how the attitudes and knowledge of students in grades K-12 impact group productivity.

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BELIEFS AND PRACTICES: STUDYING THE IMPACT OF PROFESSIONAL DEVELOPMENT FOR INSERVICE MATHEMATICS TEACHERS

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The National Council of Teachers of Mathematics (NCTM, 2000, 2014) advocates a strong, engaging curriculum for all students facilitated by teachers engaged in appropriate instruction and planning. One common thread for supporting teachers is providing appropriate professional development (PD). This study examined the impact of a three-day PD on inservice teachers' understanding and implementation of the Mathematical Teaching Practices (MTPs; NCTM, 2014). Daily reflections, a targeted vignette activity, and a Q-sort activity were utilized in the PD. Results show participants were able to identify specific MTPs to target in an action plan to reflect on their practices.

The National Council of Teachers of Mathematics (NCTM; 2000, 2014) advocates a strong, engaging mathematics curriculum for all students in K-12 facilitated by teachers engaged in appropriate mathematical instruction and planning. NCTM furthers notes that the classroom teacher is the key element in providing the type of learning experiences needed for high-level achievement by all students in mathematics. Furthermore, many leaders in mathematics education contend that there is specific knowledge needed to teach mathematics effectively that involves mathematics content, mathematical content pedagogy, and other areas (Ball, Thames, & Phelps, 2008; Shulman, 1986). The Mathematical Teaching Practices (MTPs) as identified by NCTM's *Principles to Action* (2014) support educators in their understanding and implementation of this specific knowledge. Recognizing the critical role of teachers, it is imperative that appropriate professional development (PD) is provided so that they can then provide the best instruction and curriculum possible for K-12 students.

Literature Review

According to research conducted by Stigler and Hiebert (2009), a large percentage of mathematics teachers are aware of reforms advocated by NCTM and other organizations and claim to have implemented them into their classrooms. However, the Third International Mathematics and Science Study (TIMSS) and the 1999 TIMSS-Repeat reveal through video analysis from numerous American mathematics classes that the suggested reforms are not present. Even though mathematics teachers have intentions to adopt the latest research and reform movements into their classrooms, they find it a struggle, and often misinterpret reform by merely changing surface features (Knapp & Sowder, 2004; Lewis, 2014; Stigler & Hiebert, 2009). It is therefore important to make connections between theory and practice in PD (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010).

Teachers are the key to changing the way students learn mathematics (Dana & Yendol-Silva, 2003; Darling-Hammond, 1998). In addition to what and how mathematics teachers teach, it also matters what kind of PD they participate in and the support that follows (National Research Council, 2001). According to *Professional Standards for Teaching Mathematics* (NCTM, 1991), an essential factor in teachers' PD is the degree to which they "reflect on learning and teaching individually and with colleagues" (p. 168). PD is improved with continuous program monitoring, combining strategies to address diverse needs, and building cultures that sustain learning (Loucks-Horsley et al., 2010). Furthermore, PD is strengthened when using teaching-analysis tasks. Specifically, strengthening teachers' mathematical knowledge and fostering gradual change in classroom instruction are important steps to increasing and supporting student learning opportunities (Borko, Jacobs, Koellner, & Swackhamer, 2015).

One such teaching-analysis task is a personal exploration and ranking of instructional practices using a Q-sort (Stephenson, 1935). Recently, Q-sorts have been utilized to explore preschool teachers' endorsement of instructional practices (Koutsoftas, Dubasik, & Moss, 2017) and in-service mathematics teachers' ratings of MTPs as least characteristic of their teaching/least important to most characteristic of their teaching/most important (Franz, Wilburne, Wagstaff, & Polly, 2017; Wilburne, Franz, & Polly, 2016). Another such task includes utilizing vignettes to analyze classroom cases. Multiple studies utilize vignettes in PD exercises with inservice teachers (Ambler, 2012; Angelides & Gibbs, 2006; Jeffries & Maeder, 2005; Jochums & Pershey, 1993). Engaging in a targeted vignette activity sequence can enhance and direct attention to a specific practice, such as the MTPs.

To assess the impact of PD on inservice mathematics teachers, the researchers designed a Mathematics Teachers Academy (MTA) consisting of a three-day summer training with academic-year follow-up. The MTA had a focus on algebraic thinking in Grades 5-12; formative assessment and the MTPs were used to support that focus. The PD was designed to draw upon teachers' interests and expertise in algebra, while using knowledge of the MTPs as a lens for reflection. The following research questions were crafted to examine the impact of the MTA:

- What effects does the PD have on mathematics teacher beliefs about and use of MTPs?
- What effects does the PD have on algebra teacher efficacy?
- In what ways does the use of vignettes contribute to teacher understanding of MTPs?

Methodology

Participants

Thirty-nine inservice teachers, with an average teaching experience of 9.72 years, participated in the MTA. Of these 39 participants, 23 were middle school teachers, 15 were high school teachers, and one participant did not report a specific grade level. In conjunction with the PD activities completed by these participants, the researchers collected data from several sources related to MTA activities.

Instruments and Data Collection

Data collection included three qualitative and two quantitative instruments. Participants completed an Algebra Teacher Self-Efficacy Instrument (ATSEI; Wilkerson et al., Accepted), the Teacher Action Q-sort (Franz et al., 2017), the Vignette Activity Sequence (VAS; Wilkerson, Kerschen, & Shelton, 2018), daily reflections, and action plans.

Prior to the MTA, participants completed the ATSEI, a 36-item instrument with 28 items related to functions and eight items related to technology. This instrument has been used with inservice mathematics teachers and has been tested for validity and reliability with that group. Efficacy on the ATSEI is self-reported and measured with a Likert scale of 1-6, with 6 being a higher efficacy rating. For more information about the ATSEI, see Wilkerson et al. (Accepted).

The three-day MTA began with an overview of *Principles to Action* (NCTM, 2014), specifically the MTPs. Participants also completed the Teacher Action Q-sort prior to MTA sessions, ranking specific teacher actions related to the MTPs from least to most characteristic of their teaching. The MTA included activities and formative assessments designed to support algebraic thinking. Technology resources were also shared and modeled, with graphing calculators, including Desmos, and Calculator-Based Rangers highlighted.

Each day, participants began with a whole-group activity engaging them in a mathematical problem that also allowed for discussions about the MTPs. Following the opening session, participants went to break-out sessions according to their grade band. Teachers of Grades 5-8 and teachers of Grades 9-12 were grouped separately; however, eighth grade Algebra I teachers could choose to work with either group. Participants attended at least one daily session focused on the use of a manipulative, such as algebra tiles, and one session on technology to support algebraic thinking. One session in the MTA included a Vignette Activity Sequence (VAS), in which teachers analyzed a vignette targeting three of the MTPs: Use and connect mathematical representations (MTP #3), Pose purposeful questions (MTP #5), and Elicit and use evidence of student thinking (MTP #8). As part of the VAS, participants completed a vignette activity recording sheet, providing responses about the MTPs in the vignette, making connections to their own teaching practices. For more information about the VAS, see Wilkerson et al., 2018.

At the close of each day, there was a whole-group meeting where participants engaged in a new, additional formative assessment. They also worked on setting goals as part of their action plans related to incorporating MTPs as part of their own teaching practices. The action plans were developed by the researchers and were based on SMART goals. Participants identified two to three ***S***pecific goals, how they would be ***M***easured, ***A***ctivities or steps to take to accomplish the goals, list the ***R***esources and/or who was ***R***esponsible to accomplish aspects of the goals, and a ***T***imeline for achieving the goals. Further, they were to describe potential barriers and how they might share their goals and results with administrators and other colleagues. At this time, participants completed a daily reflection consisting of a Likert scale from 1-5 to evaluate usefulness and relevance of each session attended that day. They were also asked to indicate (1) a personal goal based on experiences in sessions that day, (2) what they would take back and implement in their classrooms, (3) what the researchers could do to improve the experiences of the participants, and (4) add any additional questions or comments.

Analysis

The researchers employed a mixed methods approach using the five data sources to analyze the impact of the MTA. Analysis included an independent samples *t*-test of the ATSEI responses, analysis of Q-sort data, and axial coding of participants' VAS recording sheet, daily reflections, and action plans. The qualitative data collected in the study was coded, organized into themes, and triangulated with multiple investigators and the previously described data sources in order to corroborate the evidence based on the literature (Creswell, 2013).

Findings

During the MTA, teachers were able to reflect daily on the PD activities and how they related to the MTPs. Results are shared in the following sections based on the research questions.

What effects does the PD have on mathematics teacher beliefs about and use of MTPs?

Initial findings indicate that participants were beginning to make sense of the MTPs and examining ways they could explicitly target them in their own practice. This was found in multiple data sources, including the daily reflections, action plans, and Q-sort analyses. Analysis of the daily reflections revealed three emerging MTPs focused on by participants: Implement tasks that promote reasoning and problem solving (MTP #2), Facilitate meaning mathematical discourse (MTP #4), and Support productive struggle in learning mathematics (MTP #7).

Findings indicated all but one action plan had participants who identified goals focused on the MTPs. Some also indicated specific student mathematical practices as well. Most participants developed goals around four of the MTPs: Implement tasks that promote reasoning and problem solving (MTP #2); Use and connect mathematical representations (MTP #3); Facilitate meaningful mathematical discourse (MTP #4); and Elicit and use evidence of student thinking (MTP #8). Coding also revealed that many of the goals were related to professionalism, real-world problem solving, and use of technology. Later in the academic year, the action plans will be revisited so that participants can make decisions regarding revising their goals or establish new ones.

The Q-sort required participants to sort specific teaching actions from what was least characteristic of their teaching to most characteristic. This important reflective activity caused them to consider what they wanted their actions to be in contrast with what their actions truly were. The Q-sort revealed that many participants found Facilitate meaningful mathematical discourse (MTP #4) to be least characteristic of their teaching, while Support productive struggle in learning mathematics (MTP #7) was most characteristic of their teaching. This seems to align with the results from the goals set by participants in their action plans. The Q-sort activity will be implemented again in the next PD session in order to examine any changes or shifts in focus.

What effects does the PD have on Algebra teacher efficacy?

Results from the ATSEI survey revealed that participants self-reported higher efficacy levels for what they believed they understood about algebra compared to what they felt they could help students understand. Participants reported the lowest knowledge and teaching efficacy ratings for the four question pairs related to technology with an average rating of 2.91 on a scale of 1-6. The researchers plan to address the lowest scoring items in follow-up PD sessions and will readminister the ATSEI to measure any potential impact.

In what ways does the use of vignettes contribute to teacher understanding of MTPs?

The vignette targeted three specific MTPs: Use and connect mathematical representations (MTP #3), Pose purposeful questions (MTP #5), and Elicit and use evidence of student thinking (MTP #8). Seventy-nine percent of participants identified MTP #5 as being modeled in the vignette. The second most identified MTP was Implement tasks that promote reasoning and problem solving (MTP #2), with 66% of the participants selecting this practice. This was not one of the three MTPs targeted by the creators of the vignette. Furthermore, limited evidence was provided to support the connection between the vignette and this particular practice. However, participants who did attempt to provide evidence gave surface-level justifications. One example of this was when participants reported "Tank task assigned to group" as evidence for MTP #2 with little explanation on why or how it promotes reasoning and problem solving. This is similar to responses that participants gave in their daily evaluations. Comments such as, "I intend to use the activities that promote critical thinking and discussions" were often included but were lacking specific actions or connections to promoting reasoning and problem solving.

Many teachers indicated there was power in completing the VAS because it allowed them to better understand how important it is to ask good questions and let students struggle to find their own answers. One participant indicated the "activity is rich with new learning as well as review. This one activity covers: solution to system, domain and range, rate of change, y and x intercepts with real world meaning, and different representations." Several participants also shared that the vignette allowed them to notice valuable steps they often skip in their classroom practices, such as checking for understanding by asking peers to restate one another's justification about reasonable solutions, reflecting on student learning, and allowing students to explore in order to fully understand a problem. While it was evident in the analysis of the vignette activity that participants were able to make rich connections from the activities and actions in the vignette to their own practices, due to their limited experiences with MTPs, they struggled to give evidence to support the practices they identified in the vignette.

Implications and Future Direction

Participants will engage in follow-up PD sessions mid-year, at the end of the school year, and the following summer. Additional data will be collected at those times in order to determine the impact of the MTA on teachers' understanding and use of MTPs. Follow-up PD sessions will be designed to address findings thus far. These sessions will include an additional in-depth examining of particular MTPs through vignettes, engaging in use of specific technology for teaching and learning mathematics, and reflecting on initial proposed goals and progress toward those. In the future, the researchers would like to include observation of participants teaching during the school year in order to examine more closely the implementation aspect of the MTPs as well as progress toward goals.

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ACTION RESEARCH IN UNDERGRADUATE MATHEMATICS TEACHER EDUCATION

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Undergraduates enrolled in a mathematics teacher education scholarship program are required to conduct action research as a capstone project. This process elevates teacher candidates from the role of student to a classroom professional. In addition, faculty advisors of action research projects benefit as they work with scholars to research classroom issues and find ways to improve their practice. The purpose of this manuscript is to highlight the design of the ACTION program with the intent to influence others to enact a program like it at their institutions.

Program Description and Theoretical Framework

The preparation of a mathematics teacher needs to include an appropriate mix of mathematics content, learning psychology, pedagogy (including pedagogical content knowledge), and field experiences (Association of Mathematics Teacher Educators [AMTE], 2017). While there is no single method to prepare a mathematics teacher, these components are typical for teacher education programs in the country. In the summer of 2009, Bowling Green State University in Ohio instituted a program to enhance middle and high school teacher preparation in mathematics and science. Funding for student scholarships was secured through a grant from Choose Ohio First, administered by the Ohio Board of Higher Education, and the university provided cost sharing to establish the Science and Math Education in ACTION program (often referred to as simply "ACTION"). The goal was to accept approximately 25-30 students into a cohort each year, building the program to a maximum of approximately 110 students across four years. The lead author is the director of the ACTION program, and the second author has served as an ACTION advisor for nine students. We focus on one facet of the program—undergraduate research—and its connections to preparing successful mathematics teachers.

This competitive scholarship program requires an application process that consists of high school transcripts, ACT scores, and a letter of recommendation, as well as answering several essay questions and participating in an online interview. At this time, five cohorts have graduated and the program has met its maximum number of accepted students. Thus, we feel it is important to share broadly the design and successes of the program in relation to preparing future Ohio mathematics and science teachers and, in particular, to spotlight one component of the ACTION program—classroom action research conducted by teacher candidates in fulfillment of a capstone project.

Most agree that action research involves the study of a problem stemming from everyday lived experiences in the classroom and seeks to understand and/or improve upon this problem (Johnson, 2012). Benefits of teachers doing action research are numerous, including the generation of knowledge that can be applied to one's own classroom and the promotion of reflection on practice (Hine, 2013). In addition, studies on the effects of having undergraduates conduct pure research in the biological, physical, and mathematical sciences indicate that students learn how to think like a scientist and gain research as well as communication skills (Seymour, Hunter, Laursen, & Deantoni, 2004). Action research allows undergraduate teacher candidates to transition from learners who are dependent on instructors to independent professionals, thereby empowering them to make significant changes in their own classrooms (Kane, 2013). Undergraduates conducting action research learn to identify classroom problems, determine a way to measure results, as well as "to evaluate change and to reflect on methods most relevant to effective teaching" (Anderson, Nelson, & Waite, 2014, p. 5). Many have argued that the inclusion of action research projects in a teacher preparation program is essential for helping new teachers to identify and address issues that arise in their classrooms (Hine, 2013).

ACTION scholars engage in the same mathematics and mathematics education coursework and field experiences as non-ACTION students; but, ACTION scholars have several additional research and internship dimensions, as shown in Figure 1.

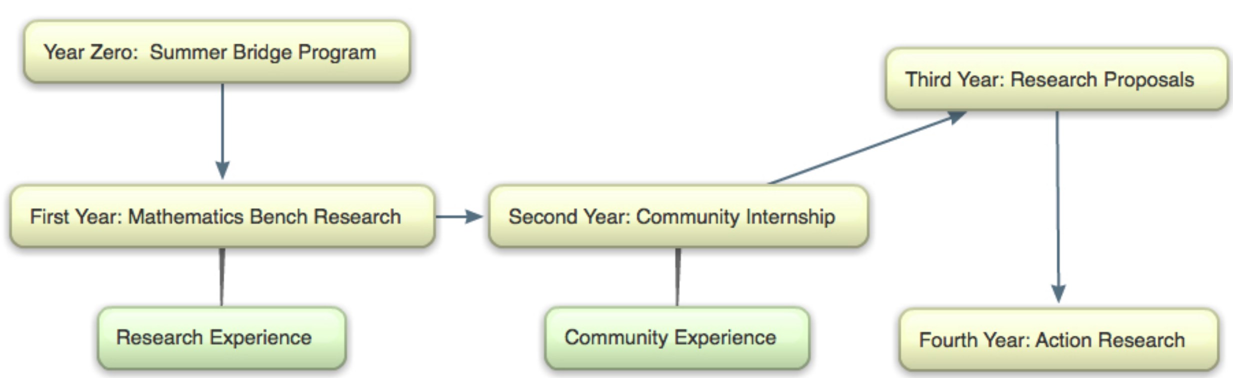

Figure 1. Components of the Science and Math Education in ACTION Program

We outline the main components of the ACTION program in the following sections.

Year Zero: Summer Bridge Program

Prior to their first year of college, students participate in a residential, four-week, Summer Bridge Program to introduce them to the professors and content areas they will be studying for the next four years. Students engage in workshops, mini-courses, and laboratory experiences with a variety of instructors, most of which are university science, mathematics, science education, and mathematics education professors.

Year One: Small Group Bench Research Projects

In the first year of college, undergraduates work in small groups (ordinarily 4-6 students per team) with a faculty member from one of the sciences or mathematics, conducting original research on a topic of the professor's choice. Undergraduates are generally surprised by this experience when conducting research alongside a university faculty member (i.e., bench research). They have reported being used to high school lab experiences in which the results of the experiments are already known and/or the investigation lacks an authentic feel of seeking to understand the world. Similarly, they are familiar with solving mathematics problems that have solutions presented in the back of the book. In research conducted at college and university campuses, there are no cookbook-style lab experiences or known solutions to the research questions and complex problems. Thus, students experience bench research in the ACTION program as they hypothesize, collect and analyze data, and draw conclusions from the results. This initial research project creates a baseline set of experiences for students' second year as novice mathematics and science researchers.

Year Two: Community-Based Internships

Undergraduates are placed in a community setting in their second year for an internship. Placements include local businesses and agencies where science and mathematics are used on the job every day. Local businesses and agencies are delighted to host university students, as they see benefits both to the students and to their organizations. Furthermore, they are also anxious to collaborate with K-12 mathematics and science teachers in ways that can help make mathematics and science seem connected to their students' everyday lives, which might include guest speakers, after school club sponsorships, and field trips. The internships are designed to help students answer the question, "Why would anyone ever need to know this?" After engaging in pure mathematics (or science) research for a year then interning in the community for another year, students are developmentally ready to think about what it means for a teacher to be a researcher in his/her own classroom.

Year Three: Defining a Problem and Writing a Literature Review

ACTION students are taught how to conduct action research in the third year. The action research is conducted the following year during their student teaching experience as a way to explore a topic of interest, collect data on it, and draw a logical conclusion aiming to improve their instruction. This third-year experience includes a yearlong course that meets monthly in which they research a topic and write a proposal for a classroom study, much like a professional learning community or lesson study in K-12 school environments. The course instruction focuses on the meaning of action research and how a teacher might act as a researcher in his/her own classroom. Students analyze research studies published in a variety of settings and learn to critique the content in terms of research design and validity of the conclusions drawn by the authors. They select a topic of interest and work individually with a faculty mentor to determine a problem. Next, they explore this area through a review of practitioner and research-based literature and write a paper describing work they intend to conduct during their student teaching experience. Faculty mentors assess their students' proposals and give feedback to them so they prepare for implementation the following year.

Year Four: Conducting Action Research

In their final year, ACTION scholars have monthly class sessions in the fall semester to closely examine appropriate qualitative and quantitative research methods for their action research. During this time, students also work individually with faculty advisors to learn about qualitative analysis approaches such as thematic analysis (Hatch, 2002) and survey analysis, as well as quantitative approaches including t-tests and chi-square analyses. The work in this course is similar to what most teachers experience in a graduate research methods class, as teacher candidates are challenged beyond the ordinary boundaries of an undergraduate education. Students finalize their proposals and then, during student teaching, conduct their action research studies, analyze their data, and write up the results in a final capstone paper.

Faculty advisors assess final capstone research papers. The intent of this paper is to use the contents to generate publications and presentations as well as to lay the groundwork for future study when new teacher begins his/her career. At a culminating senior year event, each ACTION student develops a PowerPoint and presents his/her research to an audience of peers, faculty members, and parents. This four-year experience leads to high quality action research, evaluated by practitioners and researchers at the university. The third- and fourth-year process mirrors an important facet of teachers' professional work: explore an issue in the classroom and seek to understand it and/or to solve the problem. We now turn our attention to what undergraduates and faculty mentors gain from the action research process in the program, as described through the eyes of a faculty mentor in mathematics education.

Findings and Learning Experiences

Over the past seven years, the second author served as advisor for nine ACTION students and will describe his experiences in this section. Many ACTION students have moved their undergraduate action research forward as presentations at state-level conferences or even published it in top-tier journals (e.g., *Mathematics Teaching in the Middle School*). The focus in this section is to provide first-hand experiences of an advisor—what I learned from it and what students have learned, as shared in their words.

Serving as an ACTION advisor provided several learning opportunities for me. One example is that ACTION students and I have learned more about the philosophy and methodological process for action research. Prior to serving as an ACTION advisor, I was not aware of action research process and its utility for practitioners. Numerous sessions have been offered to educate ACTION advisors about action research, including its limitations and affordances as a research approach. Action research has potential to inform mathematics educators' practice. For example, I personally engaged in action research during a mathematics education course for first-year secondary preservice teachers—collecting and analyzing data with the ultimate goal of enacting high quality instruction. Many ACTION advisees have shared that they continue to use action research as practicing teachers as a mechanism to improve their instructional practices and students' outcomes.

A second learning experience is that I have a rich opportunity to develop positive relationships with students and vice versa. Building professional relationships with students has been discussed as a key variable in retaining preservice and inservice teachers and empowering them for their career (Guarino, Santibanez, & Daley, 2006). I have become a better academic writer, reviewer of academic writing, and writing coach for others as a result of interacting with my students. Faculty members review multiple drafts of research papers that include sections such as introduction, literature review, methodology, results, and discussion. Students typically experience the same growth while writing their action research papers. A former ACTION advisee shared, "Because of the ACTION research project, I can better present my ideas in a more academic style of writing and communicate more effectively to the mathematics education field – including my peers who are teachers." Personally, I am better able to sense coherence and flow within students' work and my own research manuscripts, and both lead to papers and presentation that are often submitted for peer review. As such, the professional relationships developed with my students help me to become a better writer and to more effectively critique student work.

A third learning experience is the opportunity to co-explore important mathematics education questions posed in everyday practice with ACTION students. It is not plausible to investigate everything that is interesting, but working with ACTION students gives me the chance to be a part of the process and learn alongside them while they do their work. Former students' action research has explored numerous topics including but not limited to promoting students' representation use, improving problem-solving performance, and enacting classroom norms for deep, meaningful mathematics learning. Through these projects, I have become more knowledgeable of diverse research areas and outcomes from action research that feeds back into practice and related literature.

A fourth learning experience is dealing with failure alongside students. Mathematics education researchers know the challenges of a rejected manuscript or instruction that misses the mark. More specifically, action research may be problematic and not succeed as hoped and that happens often (Johnston, 2012). For instance, one ACTION scholar who aimed to change classroom norms and explore students' outcomes noticed that high school students were quite resistant to any changes. As much as she persisted, students perceived the change in norms during the middle of the academic year as not appropriate. There were many reasons for this; however, one was a lack of a unified vision between the ACTION scholar (i.e., student teacher) and her cooperating mentor teacher. While we discussed that her action research might not have been as successful as she hoped, we learned how important it is to present a shared vision with any co-teachers. Thus, her action research taught me (and her) to look for ways to grow and learn. This is a valuable lesson because effectively executed research generally leads to learning something, even when the outcomes are not desirable (Cresswell, 2012).

Conclusion/Future Research

As the process of preparing mathematics teachers for the future continues to be researched and improved, it is essential that educators consider the value of action research to enhance the undergraduate experience. Undergraduate teacher candidates need to learn to be teacher researchers. We have shown that there are many opportunities for university faculty to learn with students through their implementation of action research projects. Graduates of the program frequently report that the hiring process was facilitated by these research experiences, as they can discuss the impact of action research in job interviews. Similarly, principals report on the positive impressions made by ACTION scholars in the interviewing process, as graduates can describe classroom instructional issues and how they carefully studied their teaching practices to establish goals for improvement. Faculty advisors have also benefitted from this component of the ACTION program and find it exciting to be a part of the action research process. More research on the specific impacts of the capstone project on both the faculty advisors and the long-term development of teachers is yet to be conducted since the program is relatively new and graduates of ACTION have taught fewer than five years. However, we know that ACTION graduates' familiarity and experience with action research makes them highly desirable to school districts, as the ACTION program continues to promote students' growth as reflective practitioners. If a goal of educators is to improve K-12 students' performance in schools then teachers must continue to systematically determine ways to improve their practice, and action research is fundamental in the process of continual improvement.

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PROGRAMMATIC EFFECTS ON HIGH STAKES MEASURES IN SECONDARY MATH TEACHER PREPARATION

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This study examines the impact of program design, coursework, and specific assignments on high stakes measures that hold most programs accountable (e.g. Praxis II, edTPA, state/national standards). Our preliminary research question for this paper is: (1) What are the more highly correlated program components related to successful high stakes outcomes for teacher candidates? Initial findings indicate grades in advanced perspective mathematics courses and classroom observation scores are very important at successful completion of high stakes measures. Early interventions for lesser prepared teacher candidates in freshman/sophomore years are warranted.

An important component of a strong K-12 education system is a well-qualified, well-prepared classroom teacher. However, there is a critical shortage of secondary mathematics) teachers (SEMA) in many states and regions of the United States, pronounced in some areas for about two decades while other regions more recently (U.S. Department of Education, 2016). A distinct issue is bound to move this critical shortage to a more grave and dangerous situation. This situation threatens the future of many K-12 students' futures and the ability of schools to functionally serve society well. Many states under political and public pressure have increased the standards and requirements to enter the teaching profession by raising the cut scores on the Praxis II Mathematics Exam and instituting a newer high stakes measure, the Educational Teaching Portfolio Assessment (edTPA, 2017; ETS, 2017). Given this teacher shortage and rise in professional qualifying exams, it is imperative that SEMA teacher education programs (TEPs) reduce attrition and virtually eliminate failures on qualifying professional exams such as Praxis II Mathematics and the edTPA professional portfolio, with strong preparation and commitments to producing well-prepared first year teachers of mathematics. The focus of this ongoing study is to examine the impact of aligning a SEMA TEP as close as possible to the Conference Board of the Mathematical Sciences' Mathematics Education of Teachers II (CBMS MET II) recommendations of coursework, as well as structuring a strategic sequence of coursework which adheres to the recent release of the Association of Mathematics Teacher Educators' (AMTE) Standards for Preparing Teachers of Mathematics (SPTM) (CBMS, 2012; AMTE, 2017). In this early paper, we present our initial evaluation of one research question. We also note, the vision of this work is that teacher candidates are well-prepared first year teachers, and that high stakes outcome measures (professional licensure exams) should be a formality for teacher candidates rather than *test prep*.

Related Literature

Conference Board of the Mathematical Sciences

The CBMS is the umbrella organization comprised of 16 professional societies whose primary objectives include the improvement and gain of knowledge in the mathematical sciences. These professional groups work hand-in-hand to promote advanced research, improve educational outcomes in mathematics, and expand how mathematics can be used more effectively to improve the human experience. In 2001, these 16 organizations published the Mathematics Education of Teachers (MET I) primarily to open the discussion of the preparation of mathematics teachers and present the importance of mathematicians and statisticians in this preparation pathway. Entering the 21st century, coherent specific visions for the preparation of mathematics teachers was isolate to teacher preparation institutions. The role of mathematicians and statisticians in teacher education, particularly that of SEMA teacher education, is longstanding and extremely influential (or not) in how future teachers will instruct their students. Given the decade of research, experience, and knowledge since the MET I, the CBMS published the MET II to reiterate and improve the themes from MET I. We focus specifically on the recommendations of the MET II as it pertains to coursework, program design, and well-qualified faculty who prepare teacher candidates for the profession they seek to enter.

In the early 1980s, unproductive beliefs (a.k.a. mindsets) about the teaching and learning of mathematics began to be documented by education researchers (CBMS, 2001; Stigler & Hiebert, 1999). These unproductive beliefs were recently revisited in the National Council of Teachers of Mathematics *Principles to Actions* publication (NCTM, 2014). The sheer importance that content area professionals be well-versed in pedagogical research and have the ability to engage teacher candidates in high quality mathematical learning experiences as opposed to a single pedagogical method historically dominant; direct instruction (CBMS, 2012). Since the release of the MET I at the turn of the century, improvements have been made within mathematics/statistics departments yet we are a long way from the ideal as a nation. These longstanding traditions in the U.S. had a profound effect on many college-aged students having fixed mindsets and beliefs about teaching and learning mathematics because of how they are/were taught mathematics in college. Moving from learning only *how to do* procedures to earn high grades, has moved slowly but positively, towards learning *how, why,* and *when* to do procedures through various strategies.

To learn such mathematics at a deep level, the CBMS MET II recommended specific mathematics courses and content a teacher candidate should learn under the pedagogical practices that goes well beyond *how* to do procedures and focus on the development of deep understandings of the mathematics they will teach from an advanced standpoint. The recommendation of three mathematics courses as part of the mathematics major specifically designed to do such for future teachers, as well as confirming the sequence of mathematics pedagogical methods courses that the AMTE's SMTP recommends.

Association of Mathematics Teacher Educators

The AMTE's Standards for Preparing Teachers of Mathematics (2017) extended the CBMS MET2 recommendations to be more standards driven and strategically outlined successful program design and coursework that provides teacher candidates the ability to be well-prepared beginning teachers as opposed to just barely qualified. An undergraduate major in mathematics that includes statistics, three specifically designed mathematics content knowledge courses from an advanced standpoint that builds the ability to for depth of the mathematics teacher candidates will teach. Moreover, three specific mathematics education methods courses extends from two to three courses, the CBMS MET2 recommendation. The AMTE is clear, the professional knowledge and coursework in other professional fields are much more aligned than secondary mathematics teacher preparation programs. The AMTE (2017, p 133) states,

> "a degree in electrical engineering required 49 credit hours in coursework specific to electrical engineering, 31 credit hours of supporting content courses (mathematics and physics), and 15 credit hours in general principles of engineering. In contrast, a degree in secondary mathematics education at this university could include as few as 22 credit hours specific to mathematics education (which also includes a student-teaching experience), with 42 credit hours in mathematics courses taken by all mathematics majors and 15 credit hours in education courses taken by candidates from all teaching fields. Clearly, the balance of coursework in mathematics teacher preparation is out of alignment with other professional preparation programs. Students in secondary mathematics education programs need and deserve coursework to specifically prepare them for success in their field of study, including mathematics-specific methods courses and mathematics content courses specific to teaching as well as meaningful clinical experiences in secondary mathematics classrooms overseen by supervisors who

> have expertise in secondary mathematics across the grades they will be certified to teach;…".

Given the overwhelming literature in mathematics teacher education regarding appropriate coursework and structural design, the field needs an evidenced-based model of the impact of these commitments on the high stakes outcome measures of programs and teacher candidates.

Method

Present Study Program Design

The University of Alabama committed in 2009 to three successive mathematics methods courses before student teaching (only two prior to 2009) for both traditional certification teacher candidates as well as alternative certification master's student with a bachelor's degree. At the time, one mathematics course specifically designed for SEMA teacher candidates existed (geometry). In 2012, a second mathematics content course began through a commitment of the mathematics department to improve the mathematical content knowledge of SEMA TCs. These two courses fit the recommendations of the CBMS and AMTE focusing on advanced algebraic connections and transformational geometry/trigonometry from advanced standpoints.

Teacher candidates navigate this coursework in a sequenced designed, two-year cohort model. In Semester 1 (fall junior year), TCs complete nine credit hours of coursework specific to mathematics education: advanced algebraic connections, technology methods course for mathematics, and an introduction to secondary education for mathematics students. TCs generally are enrolled in an educational psychology and foundations course, as well as an additional mathematics course as part of the mathematics major. In Semester 2, TCs complete the second mathematics methods course on curriculum, lesson design and task development, as well as the second advanced perspective mathematics course focused on geometry/trigonometry. TCs are also enrolled in an additional general education course and one or two additional mathematics courses in the mathematics major. In Semester 3, a capstone mathematics methods course focuses on long-term unit planning and the sequential building of student learning, while also enrolled in a general assessment course for all disciplines and a content reading course (state requirement). Most candidates have completed the mathematics major to this point, though a few may have one additional mathematics course. In semesters 1-3, TCs are all placed in mathematics classrooms with a teacher mentor, sequentially moving from 40, to 60, to 100+

hours in each semester respectively. TCs enter the student teaching internship with more than 200 classroom hours, multiple observations of teaching, and diverse school experiences.

Population

The present paper presents initial analyses of the four cohorts (N=52) performance on high stakes mathematics and teaching portfolio exams since our 2012 implementation of the second advanced perspective mathematics course. The year 2012 marked changes to the Praxis II Mathematics exam by ETS to align with the National Council of Teachers of Mathematics revised SPA CAEP Standards. Given the space limitations of these proceedings, we aggregate and disaggregate the programmatic data analyses on these high stakes measures. A subsequent research paper that uses advanced statistical analyses will seek publication in late 2018. We separate TCs into two groups by using the ACT composite and mathematics subscores of 28; a very good proxy of mathematical college readiness for math majors (Zelkowski, 2011).

Instrumentation

The Mathematics Classroom Observation Protocol for Practices (MCOP^2) (Gleason, Livers, & Zelkowski, 2017) measures two factors that includes a measure on a mathematics teachers' ability to facilitate mathematical learning and practices in their classrooms. This instrument focuses on the teacher's ability to facilitate (TF) student engagement (SE) in the Standards for Mathematical Practice (SMPs) and the actual engagement of students in such practices related to improving student learning (NCTM, 2014). Before TCs enter the student teaching internship (Semester 4), program faculty conduct three (or more if needed) formal observations of teaching utilizing the MCOP^2 instrument. The mean TF and SE scores of the observations for each TC represents their individual scores on these two factor measures.

The Praxis II constitutes the high stakes measure of TCs content knowledge for which they study and learn in the mathematics major coursework. In the final cohort, the edTPA represents a nationally validated measure for pedagogical content knowledge for planning, enactment, and assessment of student learning. The total combined 15 rubrics from edTPA represent the score of each TC's pedagogical content knowledge. Prior to the 4th cohort edTPA scores, we used the program's predecessor Teacher Work Sample (TWS) portfolio which was used for cohorts one, two, and three. The TWS was similar in nature, scored with rubrics similar to the edTPA, and was a designed assessment to provide program feedback prior to edTPA being fully implemented. This is discussed in the limitation section.

Preliminary Findings

In this initial paper, we examine the correlational relationships between grades in the advanced perspective courses, mathematics methods courses, Praxis II scores, observations of teaching using a validated protocol, and professional teaching portfolios. Figure 1 shows the correlation between the average grade in both program advanced perspective mathematics and the Praxis II passing score. The range of successful TCs for ACT composite is 20-34 with a math subscore range of 19-36.

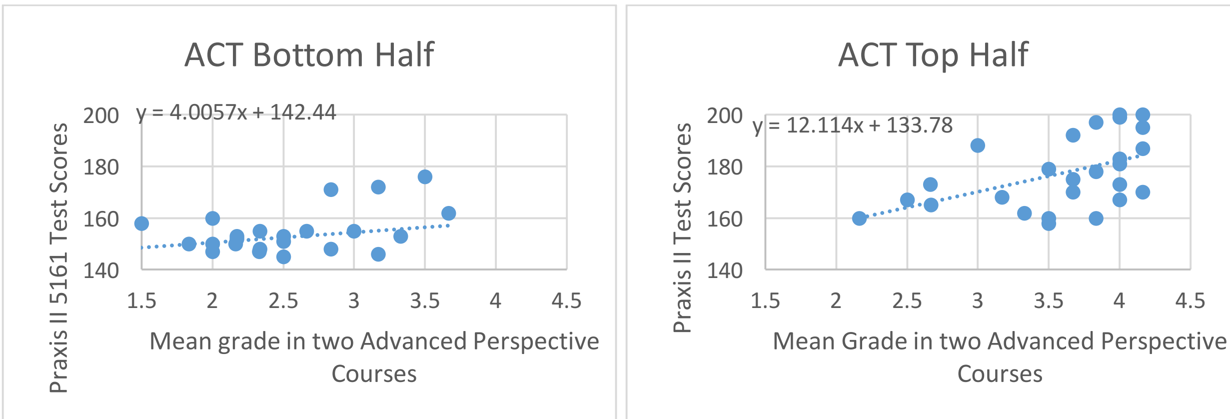

Figure 1. Correlation and model for grades in Advanced Perspective Courses and Praxis II

Figure 2 shows the correlation between the average MCOP^2 observation scores in Semester 3 with the TWS/edTPA outcome measure during semester four when TCs are fulltime teaching interns.

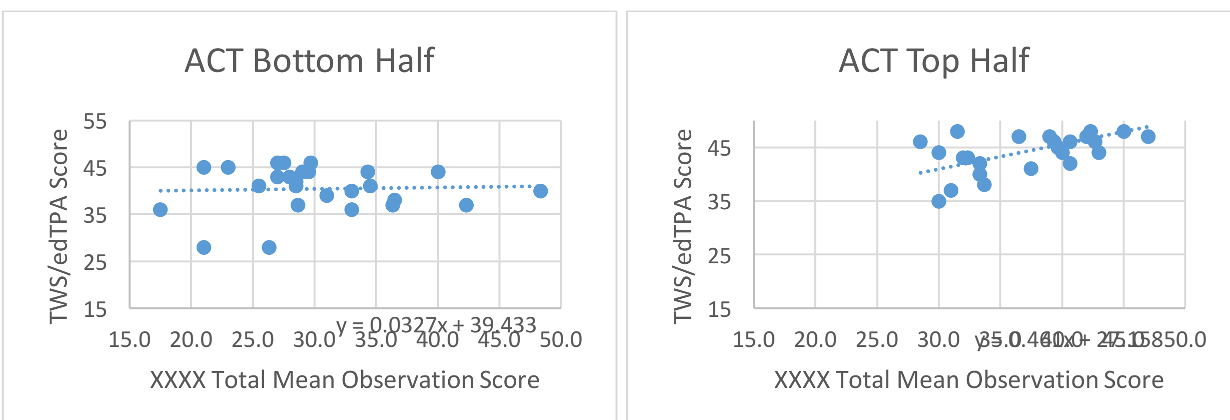

Figure 2. Correlation and model for MCOP^2 observation scores and TWS/edTPA

Our preliminary findings are one of consequence for many programs nationally. That is, the Median ACT score of 28 composite and mathematics subscore demonstrates a very different outlook for TCs on the Praxis II. Overall for the lower half, the contribution to the Praxis score from their grades in the advanced perspective mathematics courses are only 1/3 of that of the top half (i.e. four Praxis II points for each grade point v 12). Given that the current multi-state cut score for the Praxis II is 160, this means that the mean grade in two specific content courses need to be at an "A" level to make up for the lack of depth of knowledge from high school given the ACT scores. That being said, we interpret these findings to indicate early interventions are worth a closer look for TCs in the freshman/sophomore years before entering our advanced perspective courses to improve mathematical learning ability. It can also be said, that three advanced perspective courses as suggested by the CBMS and AMTE warrants serious consideration. These data at our institution has resulted in a third advanced perspective mathematics course specifically for TCs which we began in the fall 2017.

With regards to formal observations of teaching in semester three with the MCOP[2] instrument. Again, we see similar issues of concern. In general, we see only a small contribution (0.03) to overall TWS/edTPA scores for the lower half of ACT scoring TCs. The top half we see a very strong (nearly 0.50) and predictive nature of the observation scores with high-stakes outcome teaching measures during the internship. Obviously, teacher preparation programs nationally cannot institute an entrance requirement of a 28 ACT score (or comparable) and disenfranchise a large population of TCs from entering the teaching profession. However, programs should begin to identify TCs in the major in the freshman year and design learning experiences and program components to have TCs better prepared as they begin their upper division coursework.

Of concern to our findings, many teacher preparation programs nationally do not have built in *early coursework* and *experiences* during the freshman year when TCs are completing the calculus sequence and general university/college core coursework. One promising improvement worth examination is the *active learning* research action cluster (RAC) in the national Mathematics Teacher Education-Partnership (MTE-P). More information can be found here for preparation programs to begin using the work from this network improvement community of the MTE-P RAC (see http://www.aplu.org/projects-and-initiatives/stem-education/mathematics-teacher-education-partnership/mtep-racs/mtep-racs-alm.html). Improving the precalculus and calculus learning opportunities for TCs is worth strong consideration within our own program. For TCs who enter higher education lesser prepared based on ACT scores, mathematics courses that emphasize the teaching and learning of mathematics within the framework of the mathematical practices likely would improve the performance and equitable opportunities to succeed on high stakes measures.

Limitations

The outcomes of our preliminary analyses have limitations. The analyses are aggregated by two groups determined by ACT scores and do not individually demonstrate the mass achievement and growth made by individual TCs who are program completers. The national scoring of edTPA may not also align perfectly with the program's TWS assessment perfectly. The TWS had 10 rubrics and edTPA has 15. A simple conversion factor of 1.5 likely does not equate for the specificity that edTPA demands as opposed to fewer rubrics. However, the strong correlation between the upper half in both of these figures demonstrates a large impact on both the Praxis II and EdTPA measures for well-prepared college students.

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